Comparison of MOST and Bipolar transistor models

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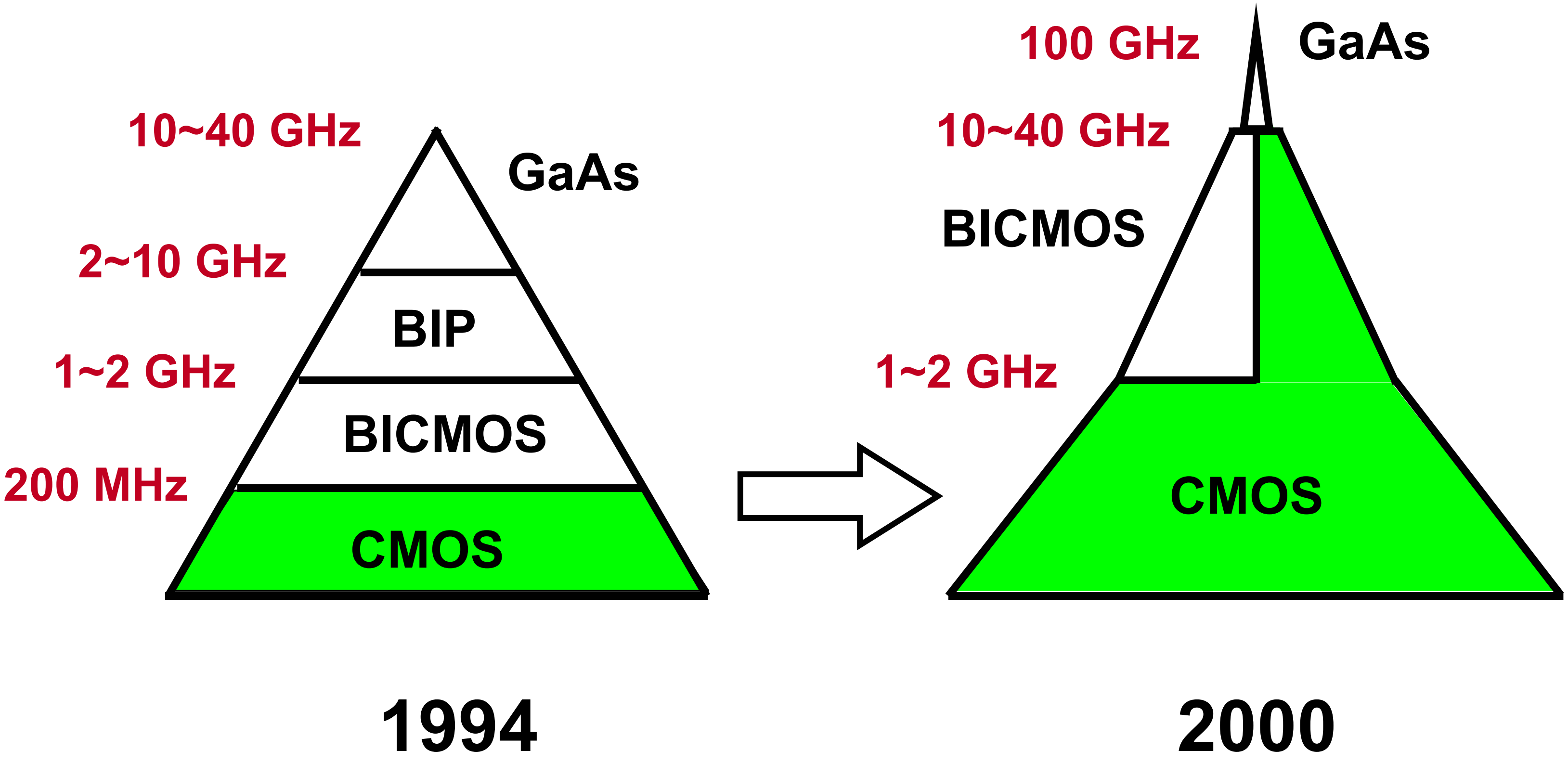
Ref.: W. Sansen : Analog Design Essentials, Springer 2006

From Bipolar to MOST transistors



Ref.Toshiba

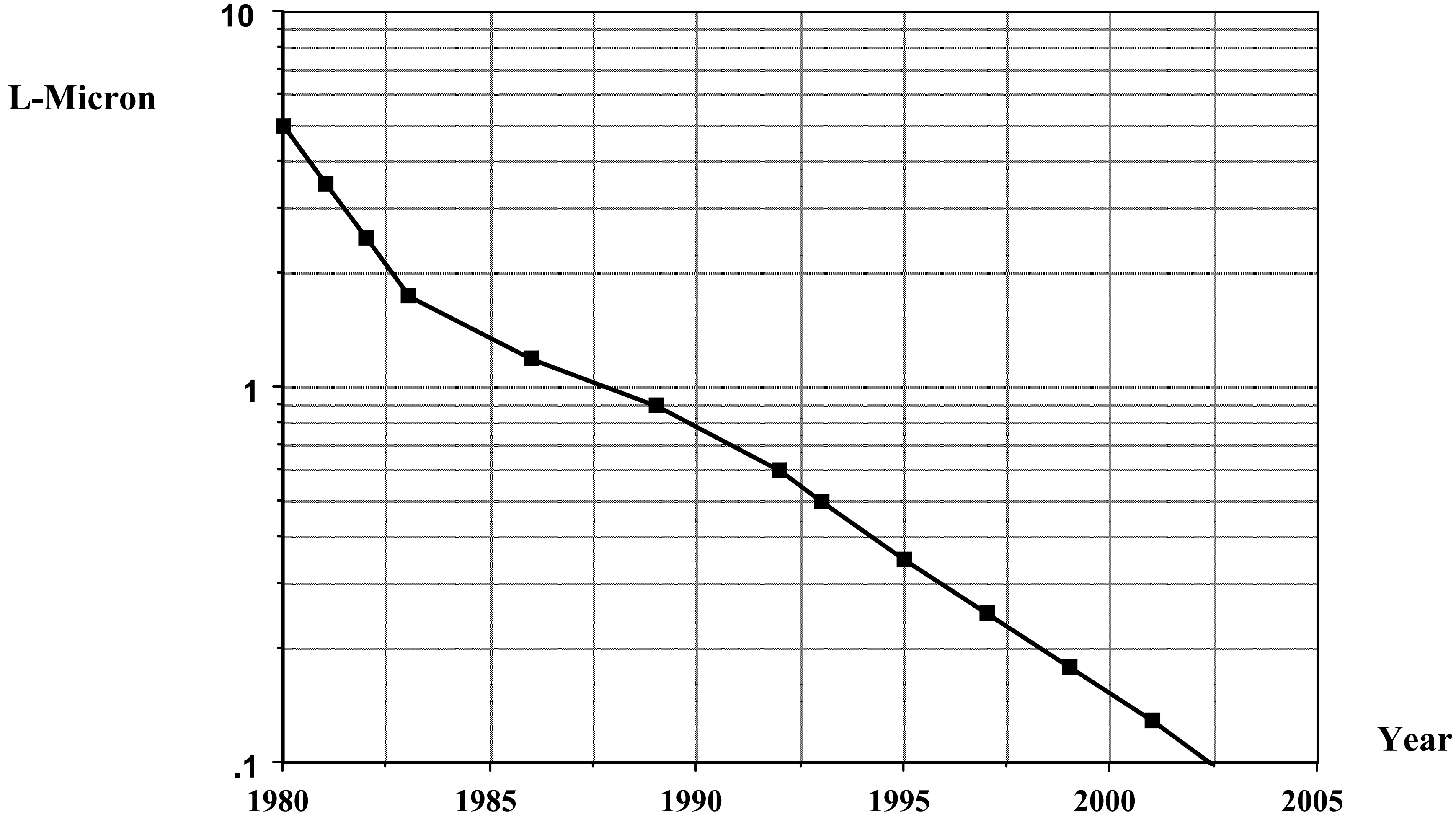
The SIA roadmap

Year	Lmin μm	Bits/chip Gb/chip	Trans/chip millions/chip	Clock MHz	Wiring
1995	0.35	0.064	4	300	4 - 5
1998	0.25	0.256	7	450	5
2001	0.18	1	13	600	5 - 6
2004	0.13	4	25	800	6
2007	0.09	16	50	1000	6 - 7
2010	0.065	64	90	1100	7 - 8

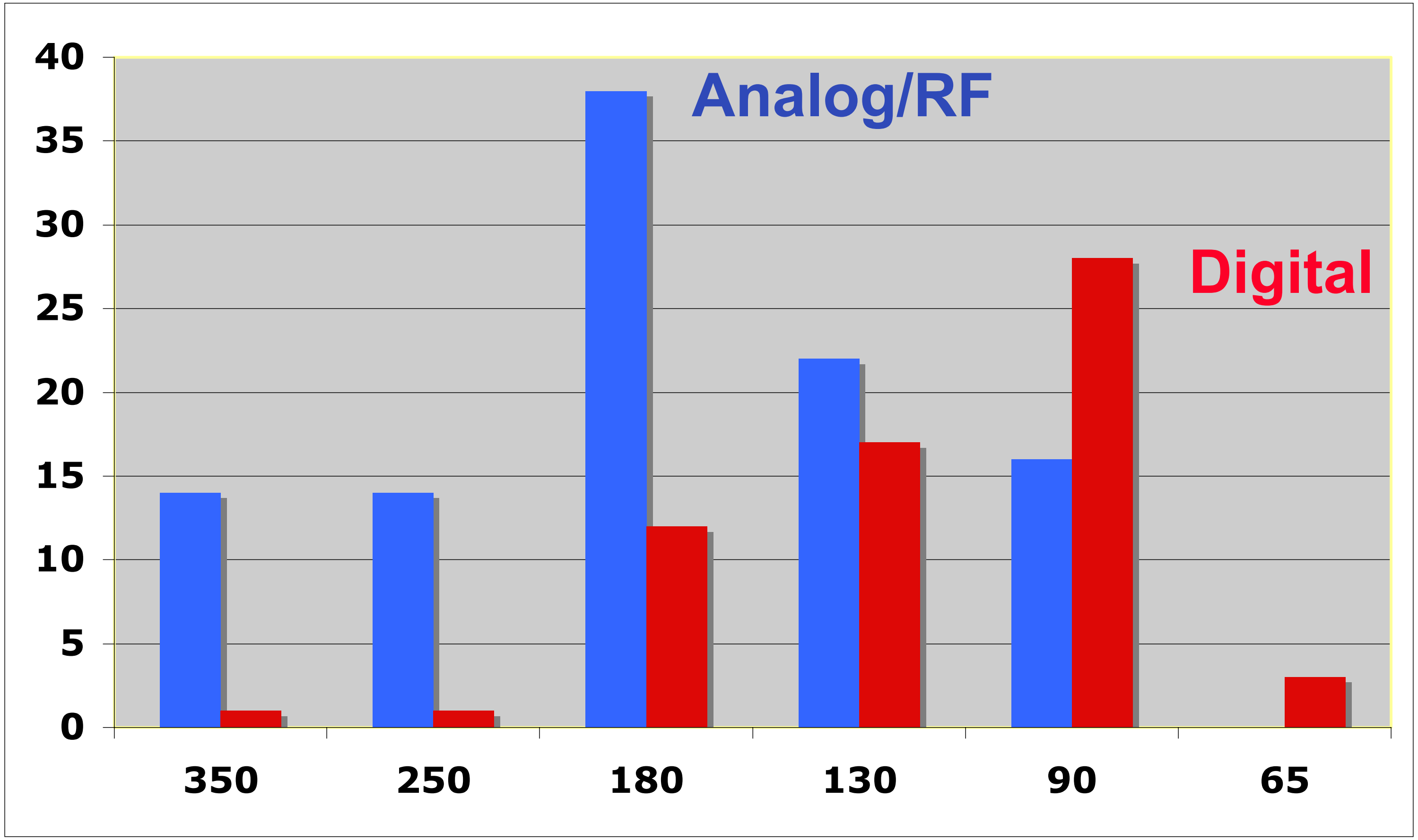
2003

Semiconductor Industry Association

The law of Moore

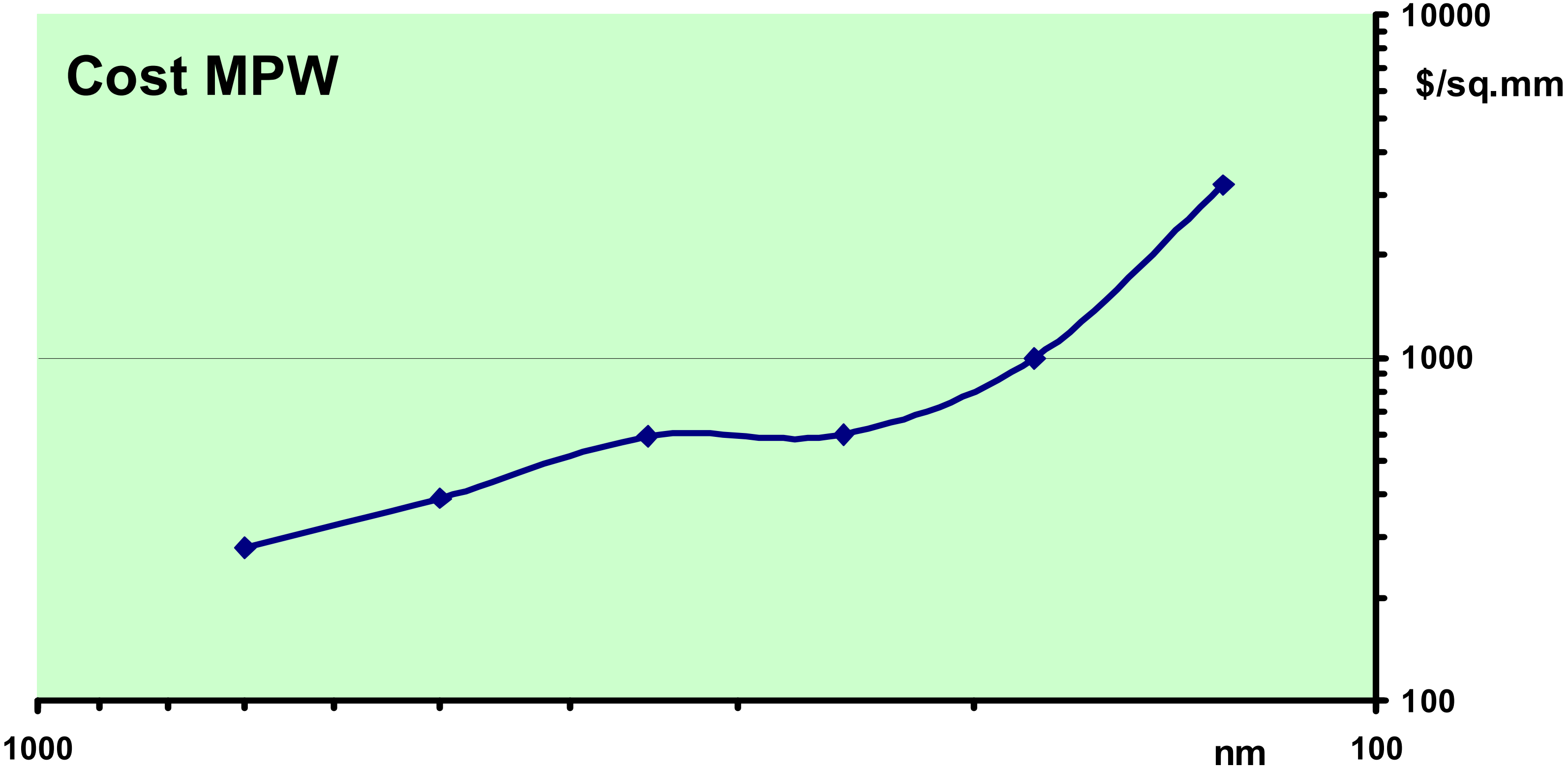


ISSCC 2005 paper distribution

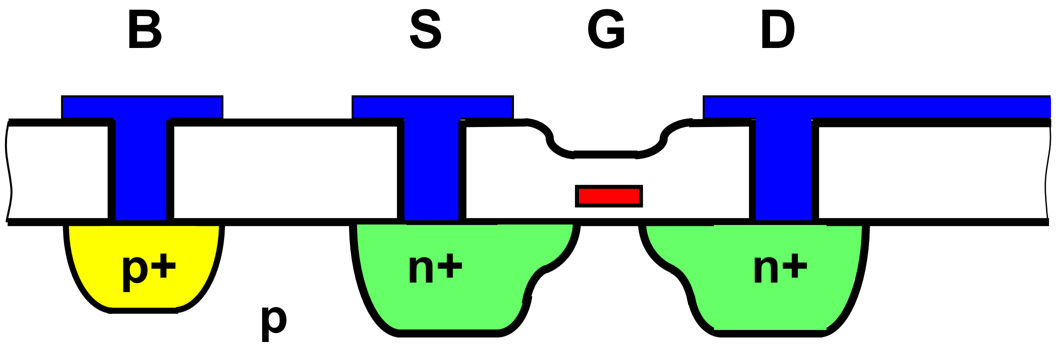


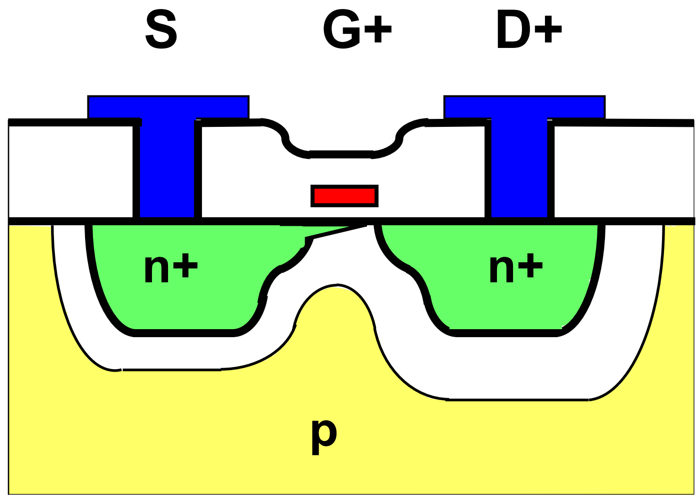
nm L_{min}

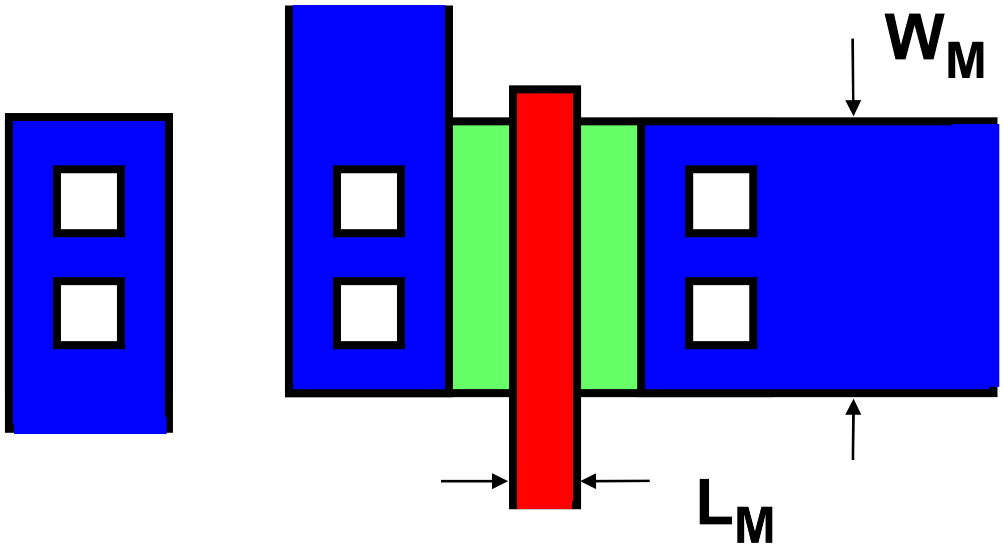
Price MPW silicon for different L (nm)



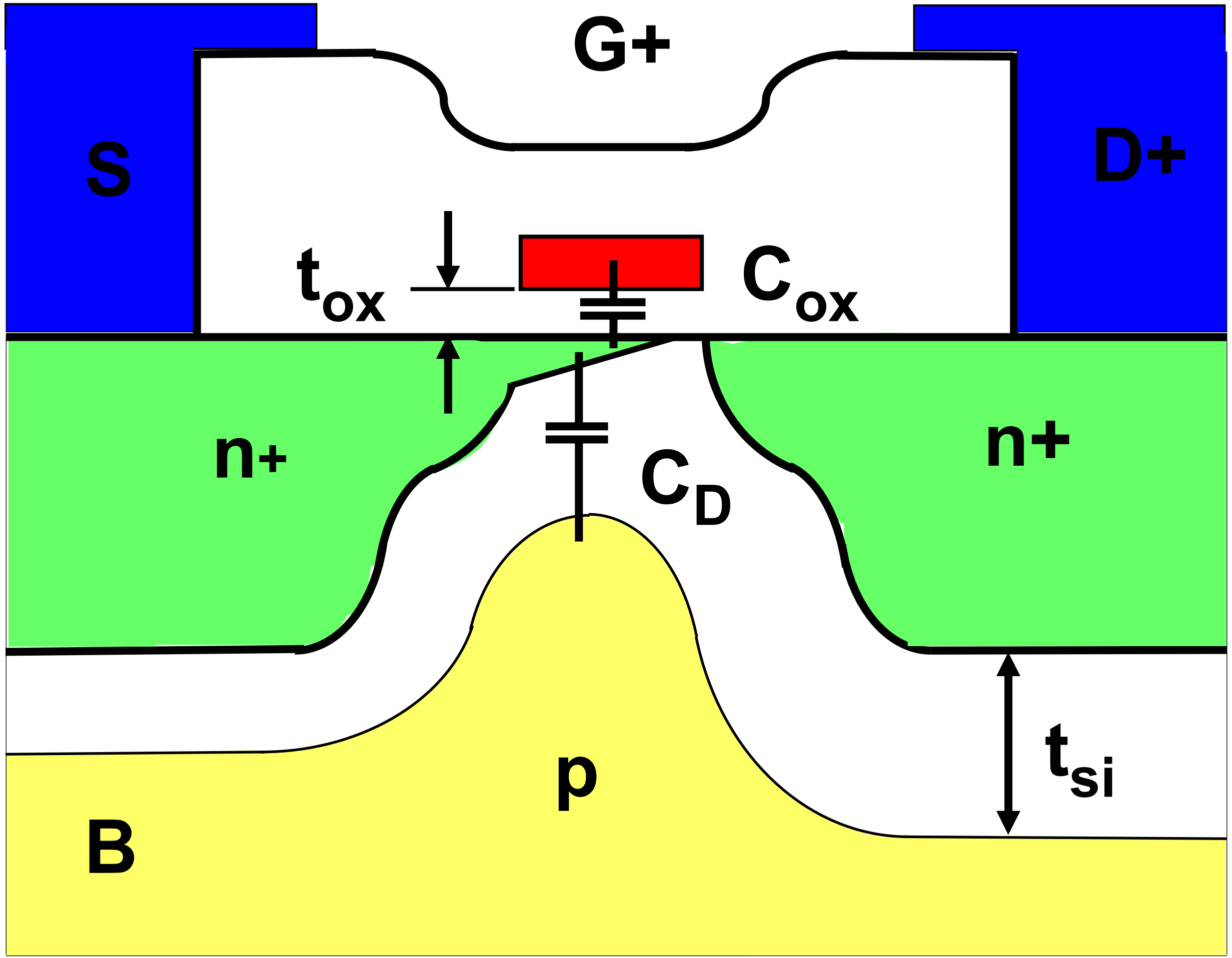
MOST layout







MOST layout : C_{ox} and C_D



$$C_D = \frac{\varepsilon_{si}}{t_{si}}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

$$\frac{C_D}{C_{ox}} = n - 1$$

MOST layout : C_{ox} and C_D values

$$C_D = \frac{\varepsilon_{si}}{t_{si}} \qquad t_{si} = \sqrt{\frac{2\varepsilon_{si}(\phi - V_{BD})}{qN_B}}$$

ε_{si} = 1 pF/cm

ε_{ox} = 0.34 pF/cm

$\phi \approx$ 0.6 V

q = 1.6 10^{-19} C

$N_B \approx 4\ 10^{17}$ cm^{-3}

Example : L = 0.35 μm; W/L = 8

V_{BD} = -3.3 V : t_{si} = 0.1 μm

$C_D \approx 10^{-7}$ F/cm^2

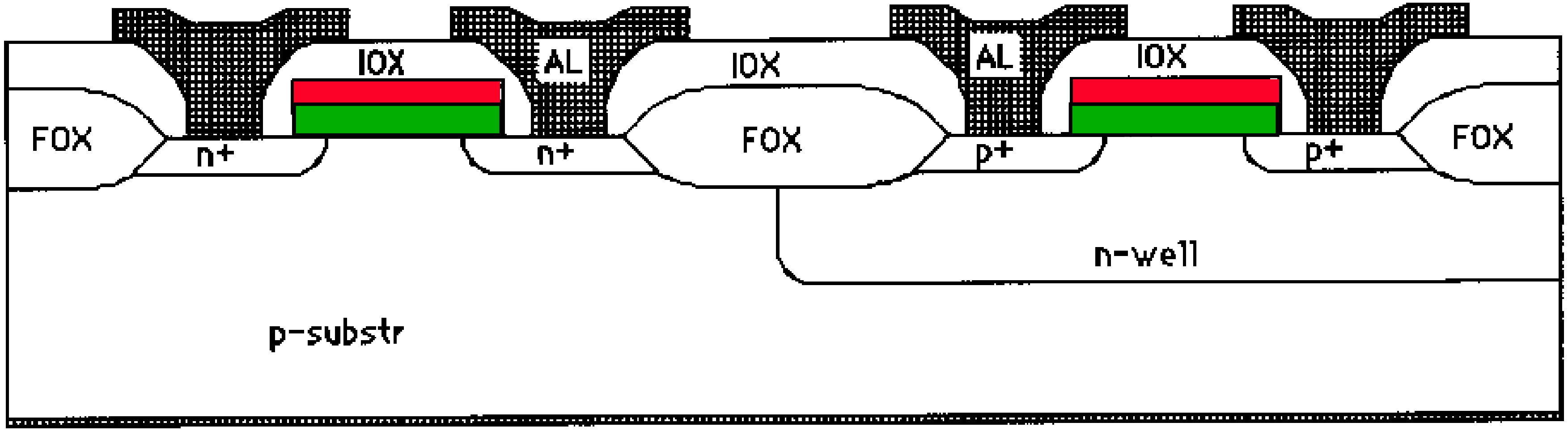
$$t_{ox} = \frac{L_{min}}{50}$$

t_{ox} = 7 nm

$C_{ox} \approx 5\ 10^{-7}$ F/cm^2

$$\frac{C_D}{C_{ox}} = n - 1 \approx 0.2$$

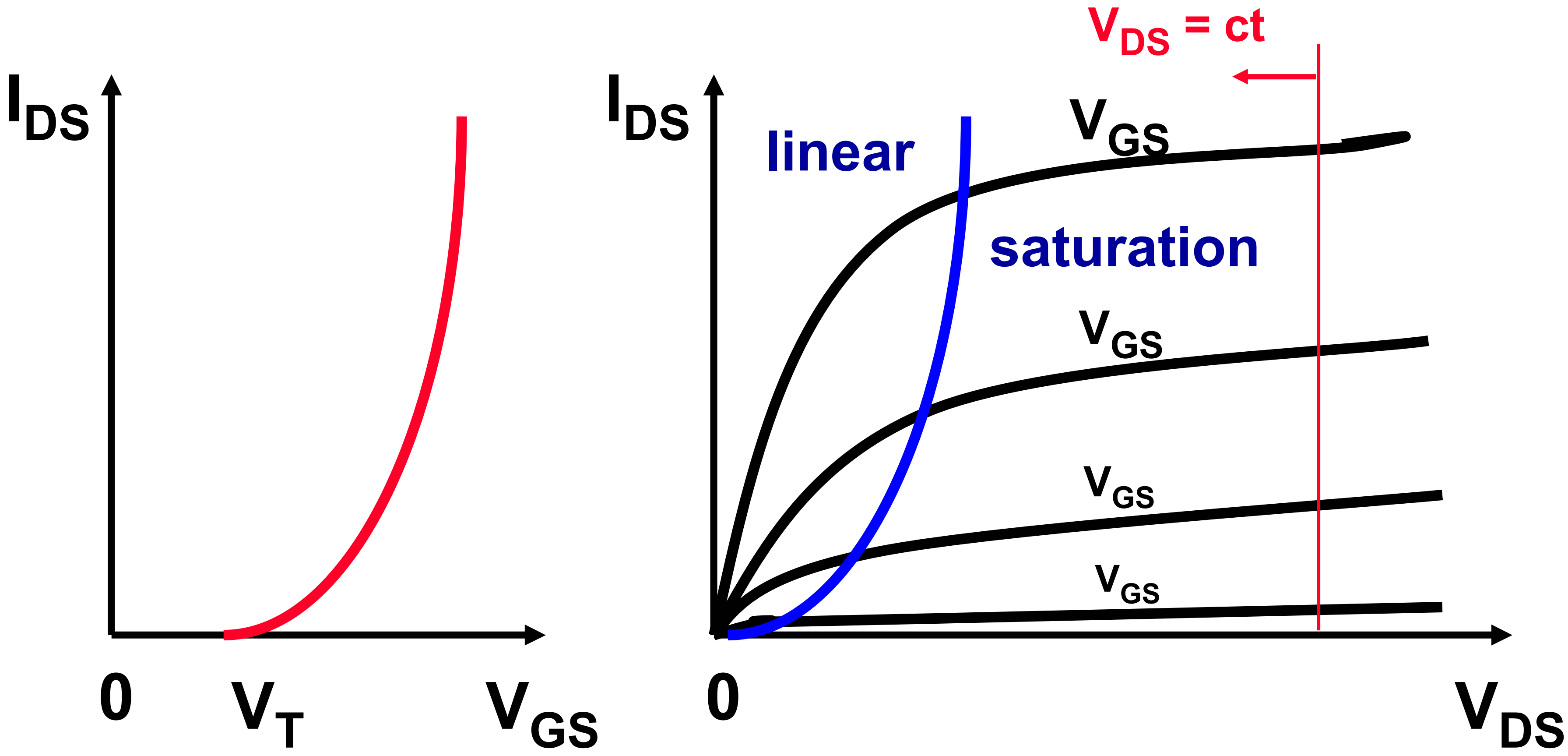
N-well CMOS technology



Gate oxyde

Polysilicon gate

MOST I_{DS} versus V_{GS} and V_{DS}



Saturation because high V_{DS}

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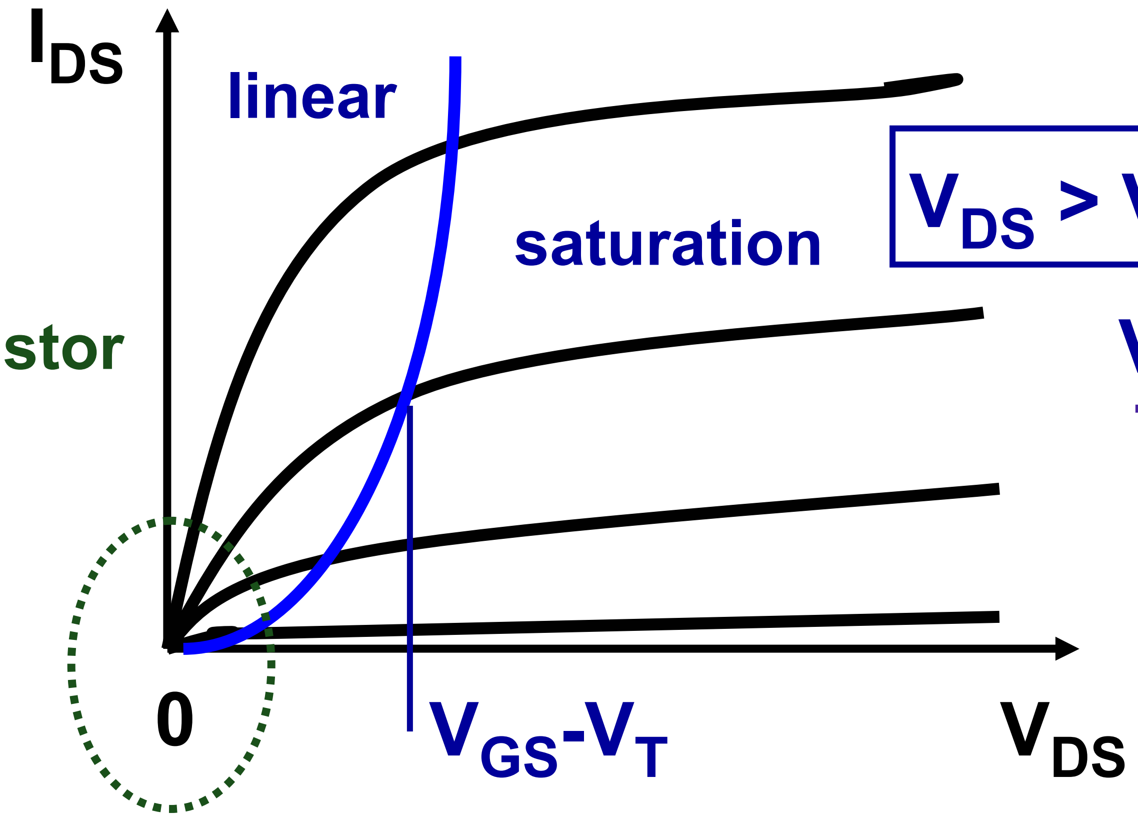
MOST I_{DS} versus V_{DS}



$V_{DS} > V_{GS}-V_T$

$\frac{V_{GS}-V_T}{n}$

Linear resistor

$R_{on} = \frac{1}{\beta (V_{GS}-V_T)}$

$\beta = KP \frac{W}{L}$

MOST parameters β , KP , C_{ox} , ...

$$\beta = KP \frac{W}{L}$$

$$KP = \mu C_{ox}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

$$t_{ox} = \frac{L_{min}}{50}$$

$KP_n \approx 300\ \mu A/V^2$

$C_{ox} \approx 5\ 10^{-7}\ F/cm^2$

$\varepsilon_{ox} = 0.34\ pF/cm$

$\varepsilon_{si} = 1\ pF/cm$

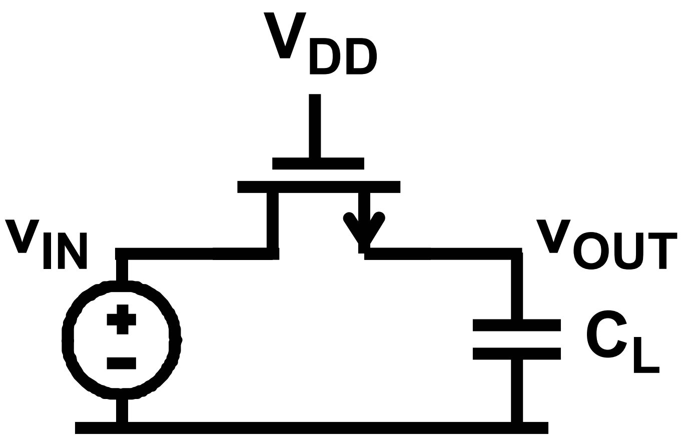
$t_{ox} = 7\ nm$

$L_{min} = 0.35\ \mu m$

$\mu_p \approx 250\ cm^2/Vs$

$\mu_n \approx 600\ cm^2/Vs$

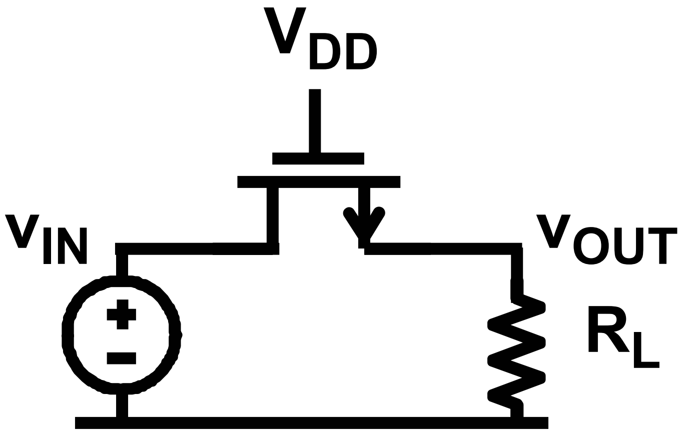
Example : Analog switch on CL



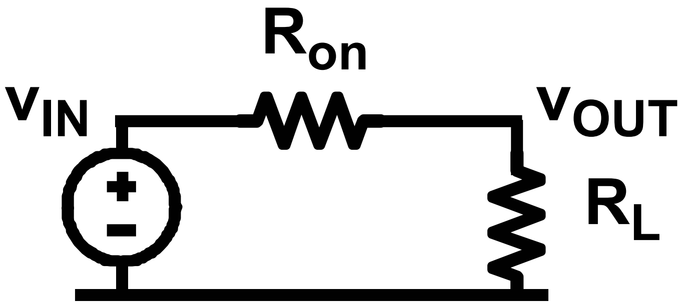
We want to switch 0.6 V to a load capacitance C_L of 4 pF.
We want to do this fast,
with time constant 0.5 ns.
Supply voltage V_{DD} = 2.5 V
V_T = 0.5 V
Use standard 0.35 μm CMOS.

Choose
minimum channel length and
find an average V_{GS} !

Example : Analog switch on RL



We want to switch 0.6 V to a load resistor R_L of 5 kΩ.
W/L = 8
Supply voltage V_{DD} = 2.5 V
0.35 μm CMOS: V_T = 0.5 V
v_{OUT} ? R_{on} ?



Choose
minimum channel length !

Body effect - Parasitic JFET

$$V_T = V_{T0} + \gamma \left[\sqrt{|2\Phi_F| + V_{BS}} - \sqrt{|2\Phi_F|} \right]$$

$$n = \frac{\gamma}{\sqrt{|2\Phi_F| + V_{BS}}} = 1 + \frac{C_D}{C_{ox}}$$

$|2\Phi_F| \approx 0.6$ V

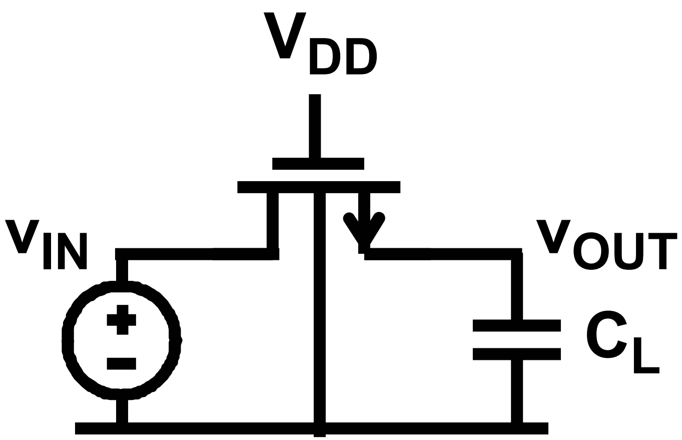
$n \approx 1.2 \ldots 1.5$

$\gamma \approx 0.5 \ldots 0.8\ V^{1/2}$

Reverse v_{BS} increases $|V_T|$ and decreases $|i_{DS}|$!!!

$n = 1/\kappa$ subthreshold gate coupling coeff. Tsividis

Ex. : Analog switch with nonzero VBS



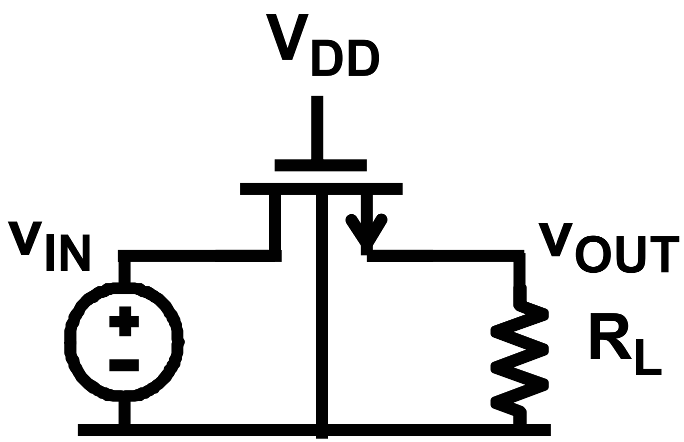
Switch 0.6 V to a load capacitance C_L of 4 pF or a load resistor R_L of 5 kΩ.

W/L = 8 (R_{on} = 125 Ω @ V_{BS} = 0)

Supply voltage V_{DD} = 2.5 V

0.35 μm CMOS: V_T = 0.5 V

v_{OUT} ? for γ = 0.5 V^{-1}



Start with V_{BS} = 0.

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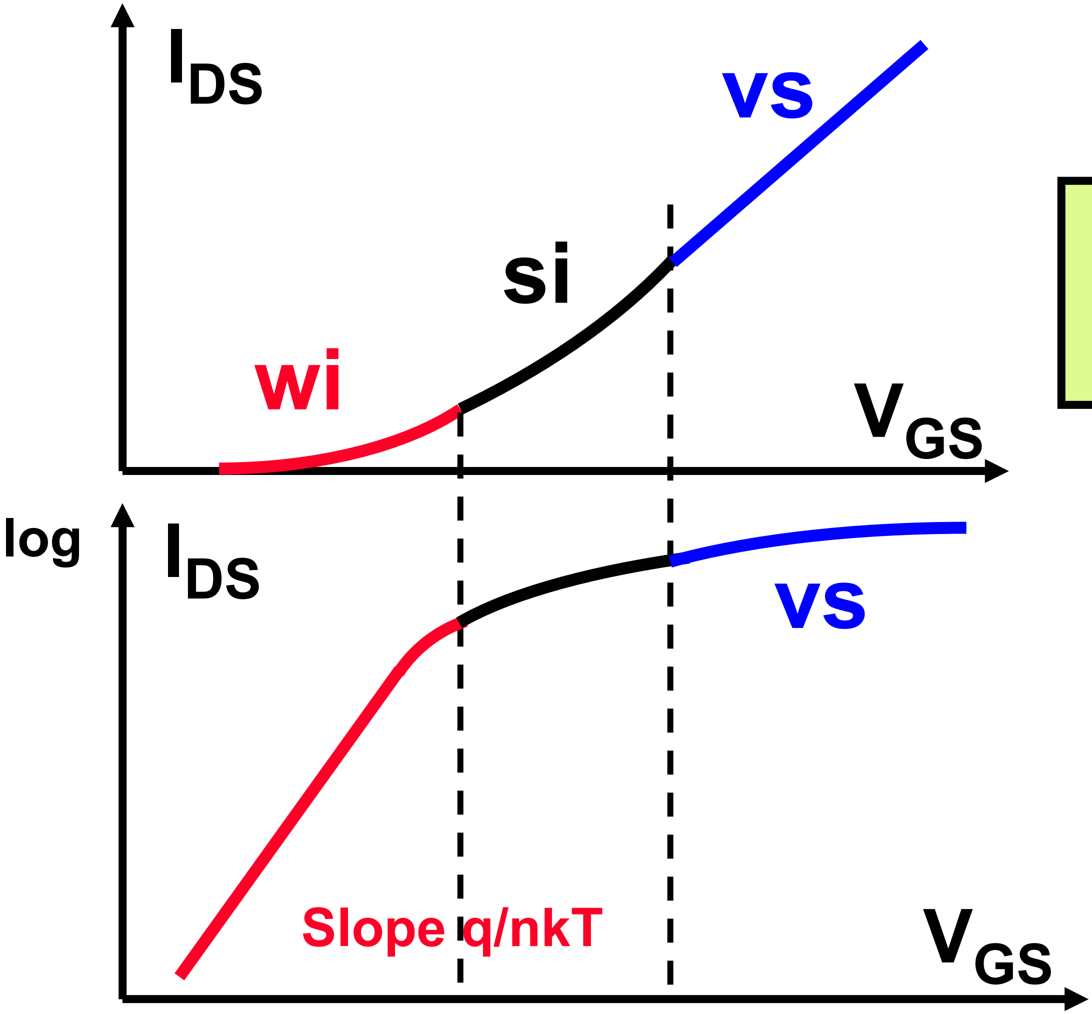
MOST I_{DS} versus V_{GS}



$I_{DS} \sim (V_{GS}-V_T)$

$$I_{DS} = K'_n \frac{W}{L}(V_{GS}-V_T)^2$$

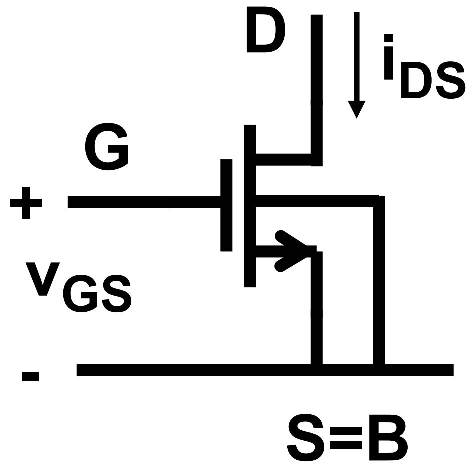
$K' = \dfrac{KP}{2\,n}$ n = ??

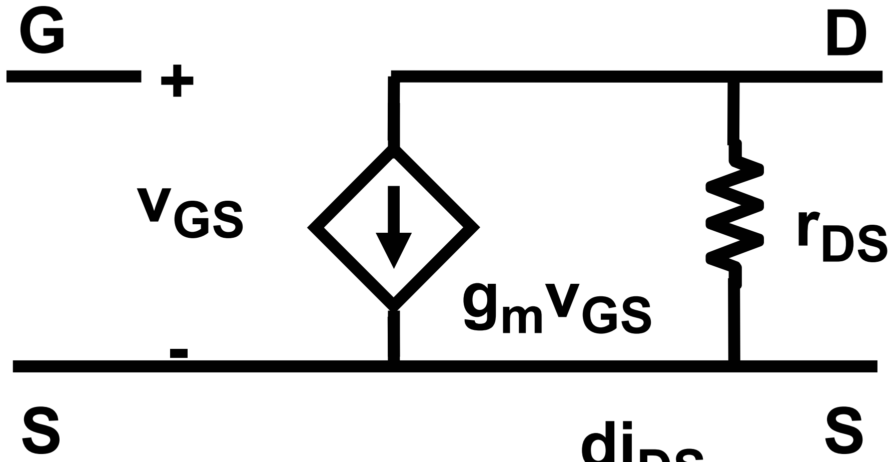
$K'_n \approx 100\ \mu A/V^2$

$K'_p \approx 40\ \mu A/V^2$

$$I_{DS} \sim \exp \frac{V_{GS}}{nkT/q}$$

MOST small-signal model : g_m & r_{DS}





$$g_m = \frac{di_{DS}}{dv_{GS}}$$

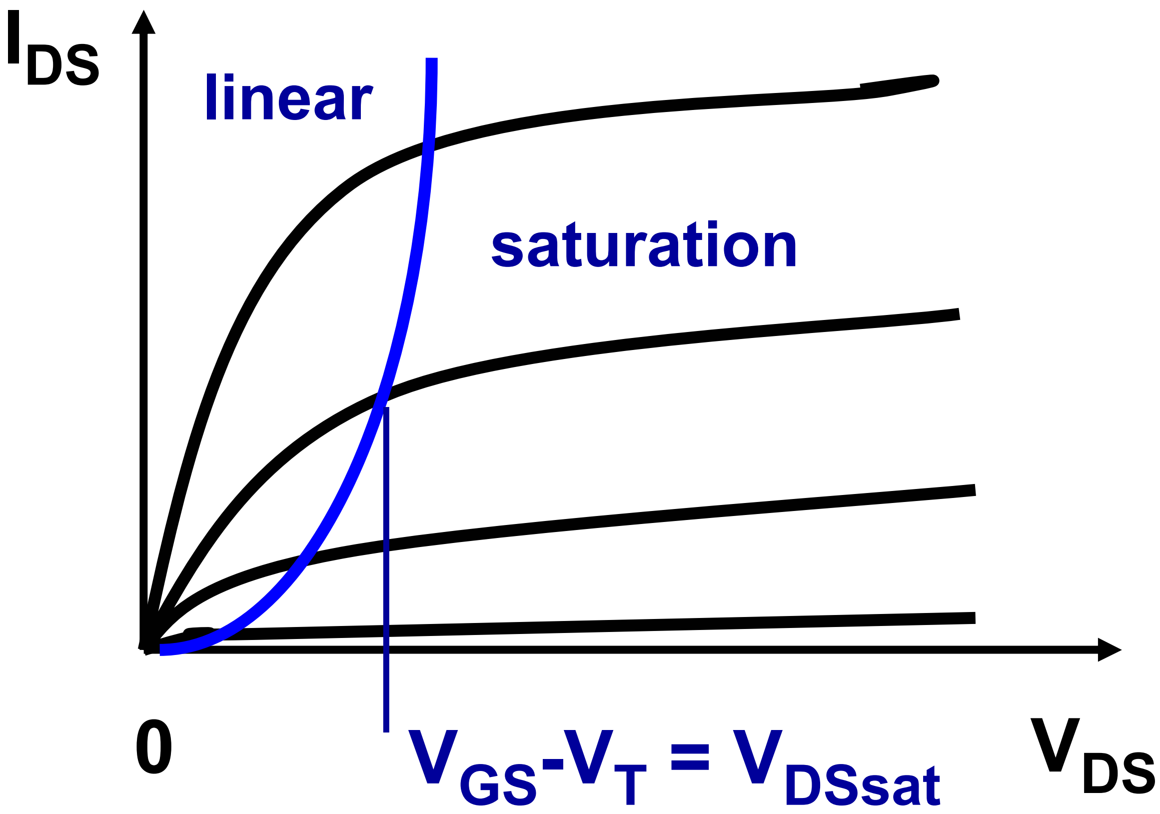
$$g_m = 2K'_n \frac{W}{L} (V_{GS}-V_T) = 2\sqrt{K'_n \frac{W}{L} I_{DS}} = \frac{2\, I_{DS}}{V_{GS}-V_T}$$

The transconductance g_m

Is $g_m \sim \sqrt{I_{DS}}$

or $\sim I_{DS}$?

MOST small-signal model : r_{DS}



$$r_{DS} = r_o = \frac{V_E L}{I_{DS}}$$

$$\lambda = \frac{1}{V_E L}$$

$V_{En} = 4\ V/\mu mL$

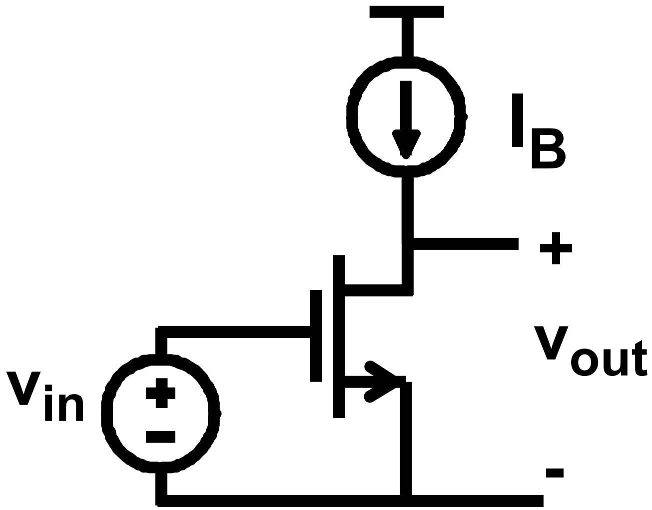
$L = 1\ \mu m$

$I_{DS} = 100\ \mu A$

$r_o = 40\ k\Omega$

$$I_{DS} = K'_n \frac{W}{L} (V_{GS}-V_T)^2 (1 + \lambda V_{DS})$$

MOST single-transistor gain A_v



$$A_v = g_m r_{DS} = \frac{2\ V_E\ L}{V_{GS}-V_T}$$

$$A_v \approx 100$$

If $V_E L \approx 10$ V

and $V_{GS}-V_T \approx 0.2$ V

Design for high gain :

	High gain	High speed
V_{GS}-V_T	Low (0.2 V)	
L	High	

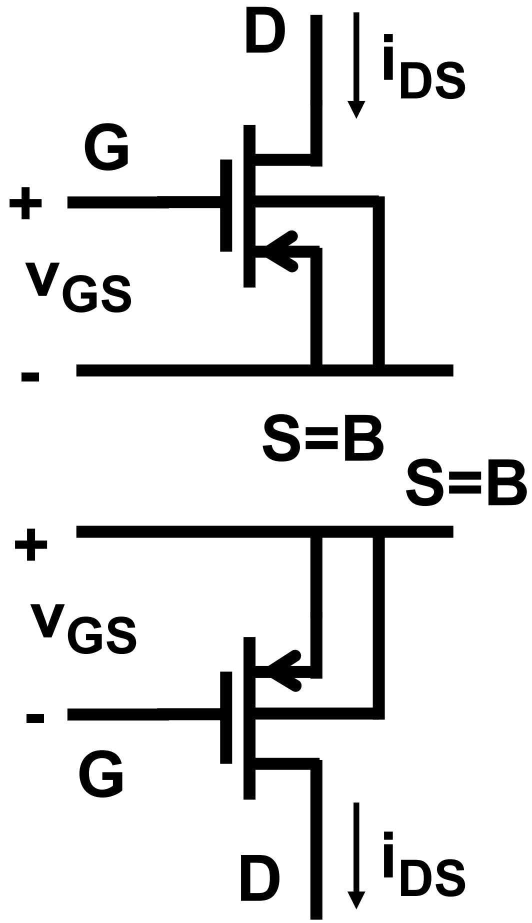
V_{GS}-V_T sets the ratio g_m/I_{DS} !

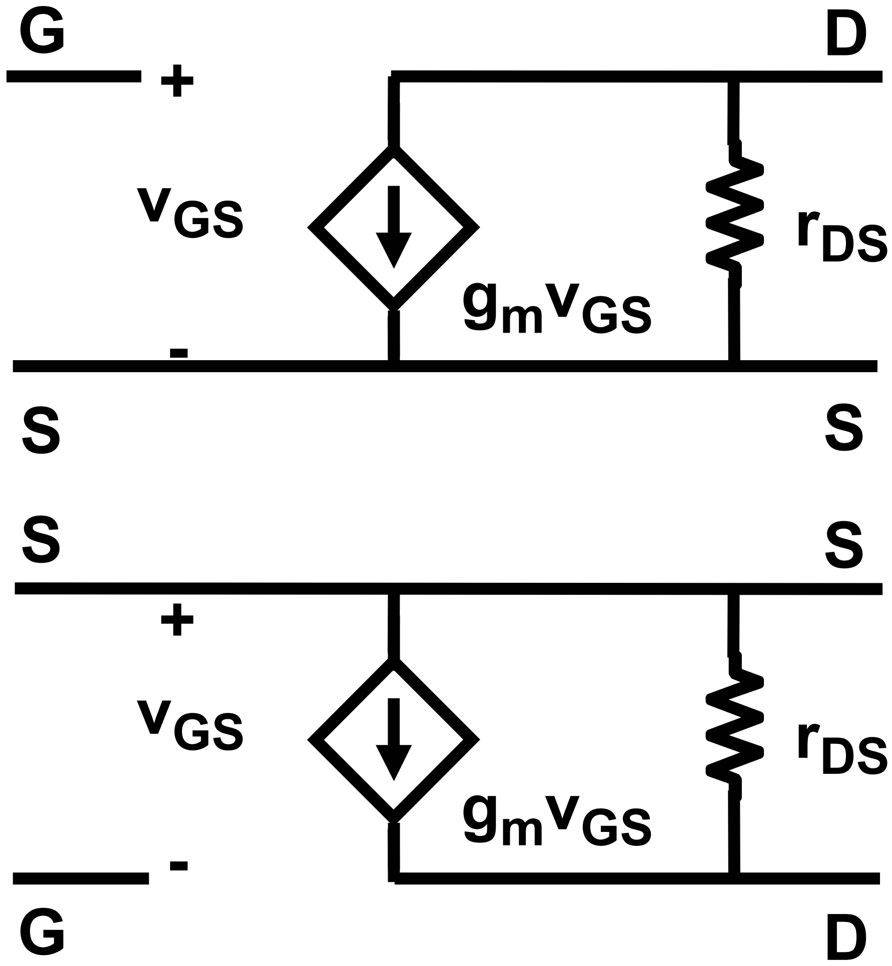
Example: single-transistor amplifier

We want to realize a three-stage amplifier
with a total gain of 10.000.
We use three single-transistor stages in series.
What minimum lengths do we have to use in
an advanced 65 nm CMOS technology
with V_E = 4 V/μm ?

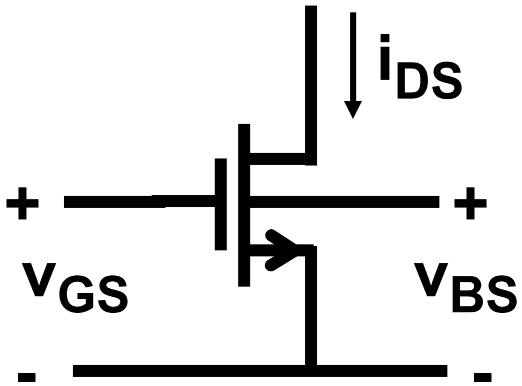
Choose
V_{GS}-V_T = 0.2 V !

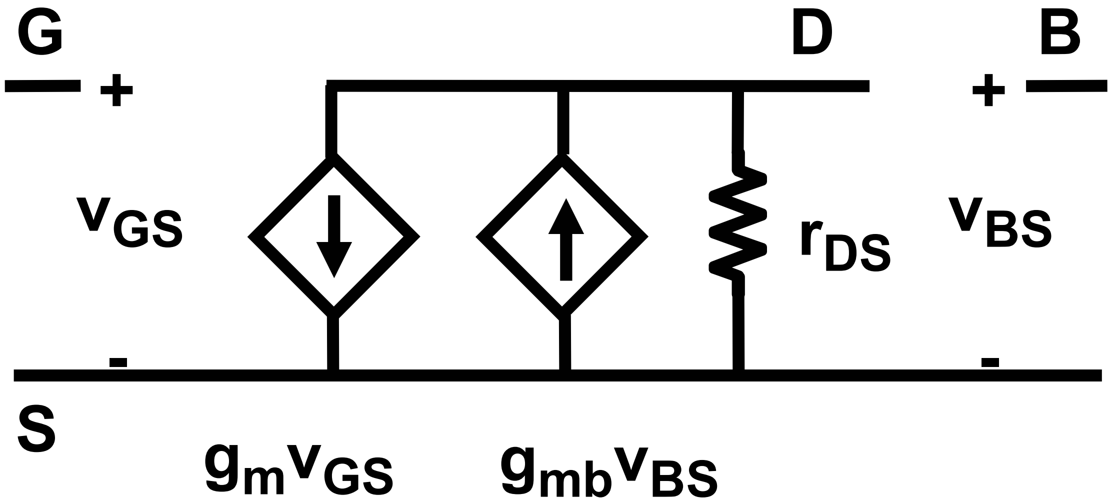
pMOST small-signal model





MOST small-signal model: g_m & g_{mb}





$$g_m = \frac{di_{DS}}{dv_{GS}} \qquad g_{mb} = \frac{di_{DS}}{dv_{BS}}$$

$$\frac{g_{mb}}{g_m} = \frac{C_D}{C_{ox}} = n - 1$$

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  - **Transition weak inversion-strong inversion**
  - **Transition strong inversion-velocity saturation**
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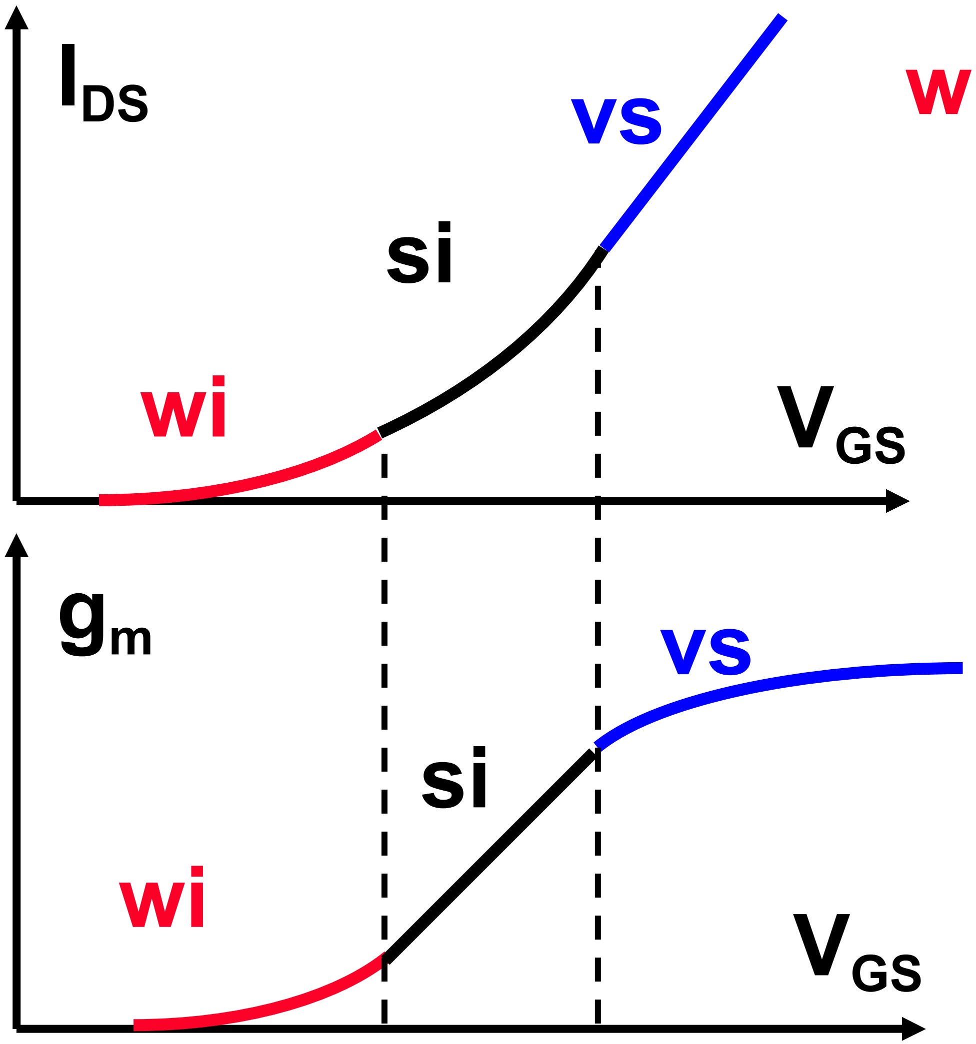
- **Models of Bipolar transistors**
- **Comparison of MOSTs & Bipolar transistors**

I_{DS} & g_m versus V_{GS} : weak inversion



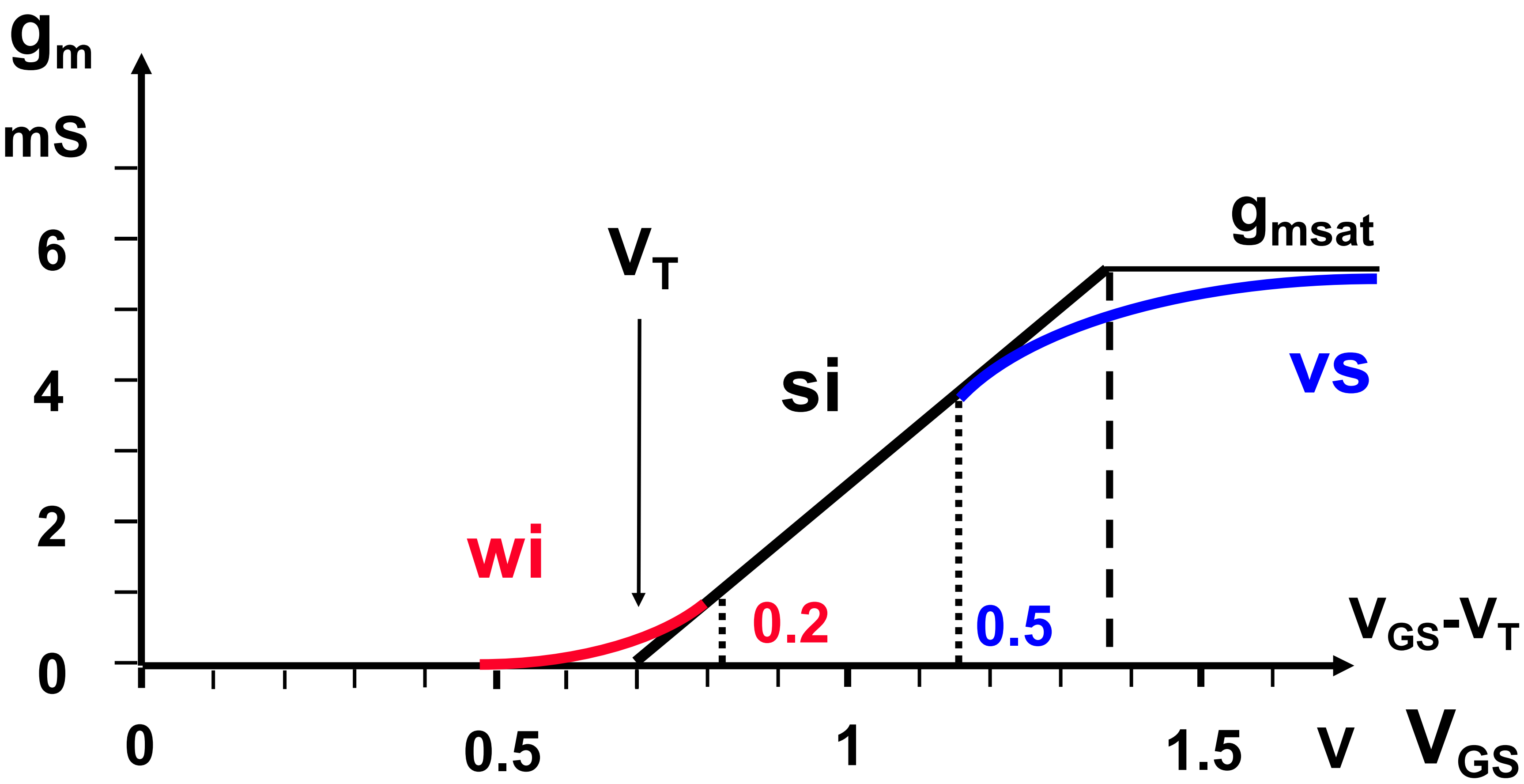
wi : weak inversion

$$I_{DSwi} = I_{D0} \frac{W}{L} \exp^{\frac{V_{GS}}{nkT/q}}$$

Subthreshold slope :
nkT/q ln(10)

$$g_{mwi} = \frac{I_{DSwi}}{nkT/q}$$

Transconductance g_m versus V_{GS}



Transition voltage V_{GSt} between wi & si

$$I_{DSwi} = I_{D0} \frac{W}{L} \exp^{\frac{V_{GS}}{nkT/q}}$$

$$I_{DS} = K'_n \frac{W}{L} (V_{GS}-V_T)^2$$

$$g_{mwi} = \frac{I_{DSwi}}{nkT/q}$$

$$g_m = \frac{2 I_{DS}}{V_{GS}-V_T}$$

$$\frac{g_{mwi}}{I_{DSwi}} = \frac{1}{nkT/q}$$

$$\frac{g_m}{I_{DS}} = \frac{2}{V_{GS}-V_T}$$

$$(V_{GSt}-V_T)_t = 2n \frac{kT}{q}$$

Transition Voltage V_{GSt} for different L

$$(V_{GSt}-V_T)_t = 2n\,\frac{kT}{q}$$

$$I_{DSt} \approx K'_n\,\frac{W}{L}\left(2n\,\frac{kT}{q}\right)^2$$

Is independent of channel length L

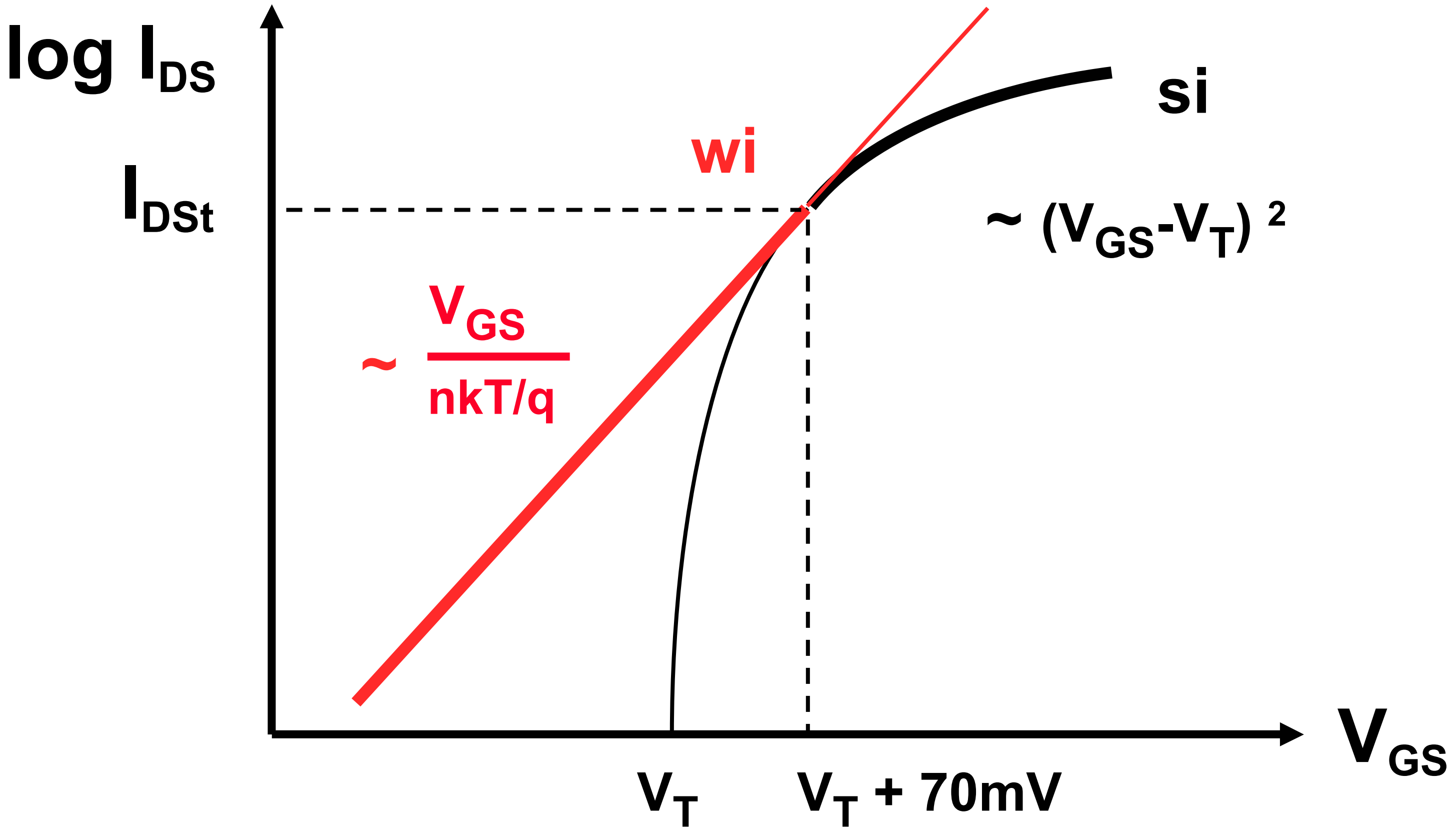
Is still true in … years !

$$(V_{GSt}-V_T)_t = 2n\,\frac{kT}{q} \approx 70\text{ mV}$$

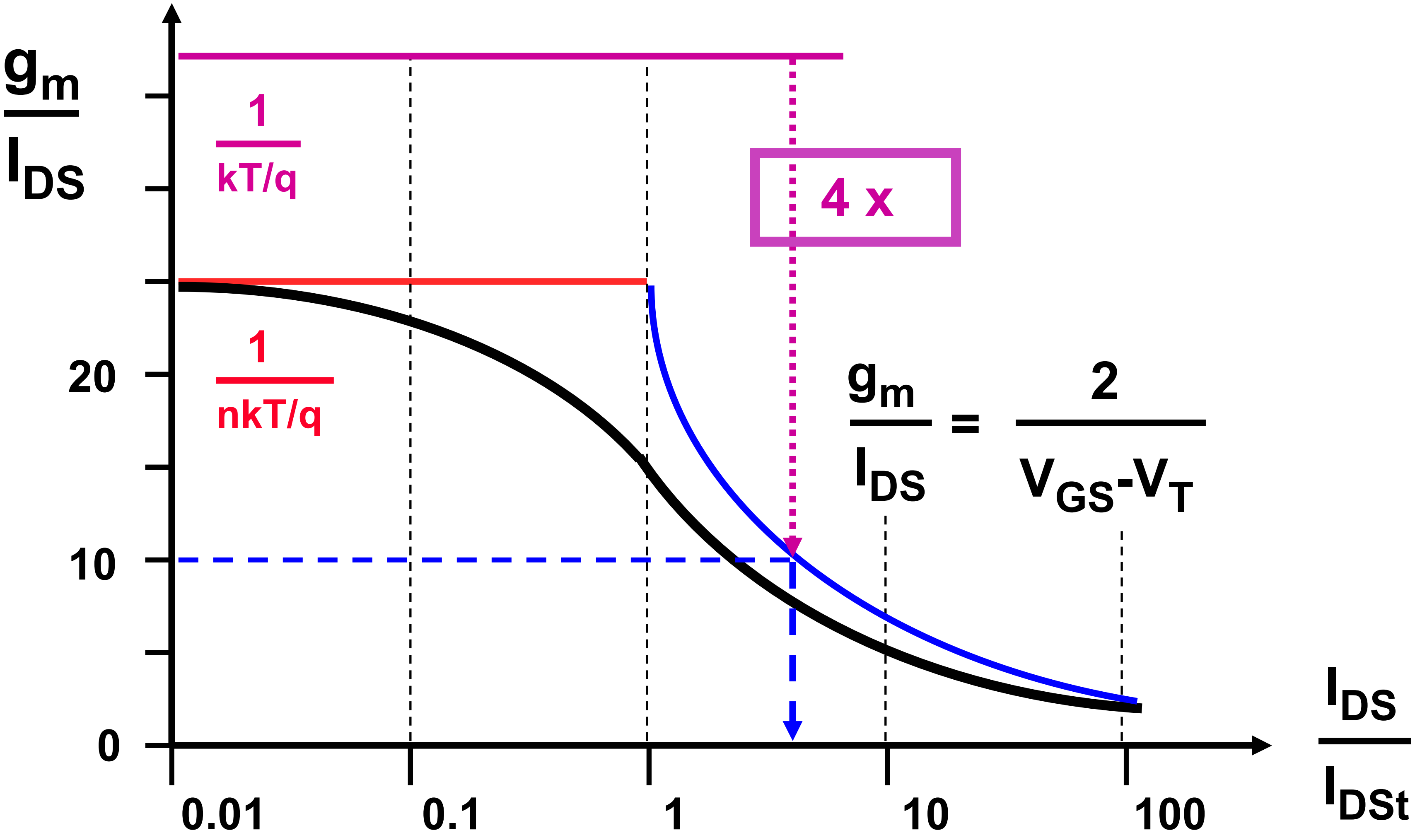
$$I_{DSt} \approx 2\ \mu A \text{ for } \frac{W}{L} = 10$$

for nMOST

Transition wi - si



Ratio g_m/I_{DS} at the transition wi - si



EKV model for smooth wi-si transition

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 [\ln(1 + e^v)]^2 \qquad V_{GST} = V_{GS} - V_T \qquad K' = \frac{KP}{2n}$$

$$v = \frac{V_{GST}}{V_{GSTt}} \qquad V_{GSTt} = (V_{GS} - V_T)_t = 2n \frac{kT}{q}$$

$$\approx 70 \text{ mV}$$

Small v : $\ln(1 + e^v) \approx e^v$

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 e^{2v} = \underbrace{K' \frac{W}{L} V_{GSTt}^2}_{I_{DSt}} \exp\left(\frac{V_{GS} - V_T}{n\, kT/q}\right)$$

Large v : $\ln(1 + e^v) \approx v$

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 v^2 = K' \frac{W}{L} (V_{GS} - V_T)^2$$

Enz, AICSP '95, 83-114
Cunha, JSSC Oct.98 1510-1519

Transition current I_{DSt} between wi & si

$$I_{DSt} = I_{DS}\Big|_{\substack{V=1\\ i=1}} = K' \frac{W}{L} V_{GSTt}^2$$

$I_{DSt} = 2\ \mu A$ for $W/L = 10$

$$i = \frac{I_{DS}}{I_{DSt}} = [\ \ln(1 + e^{v})\ ]^2$$

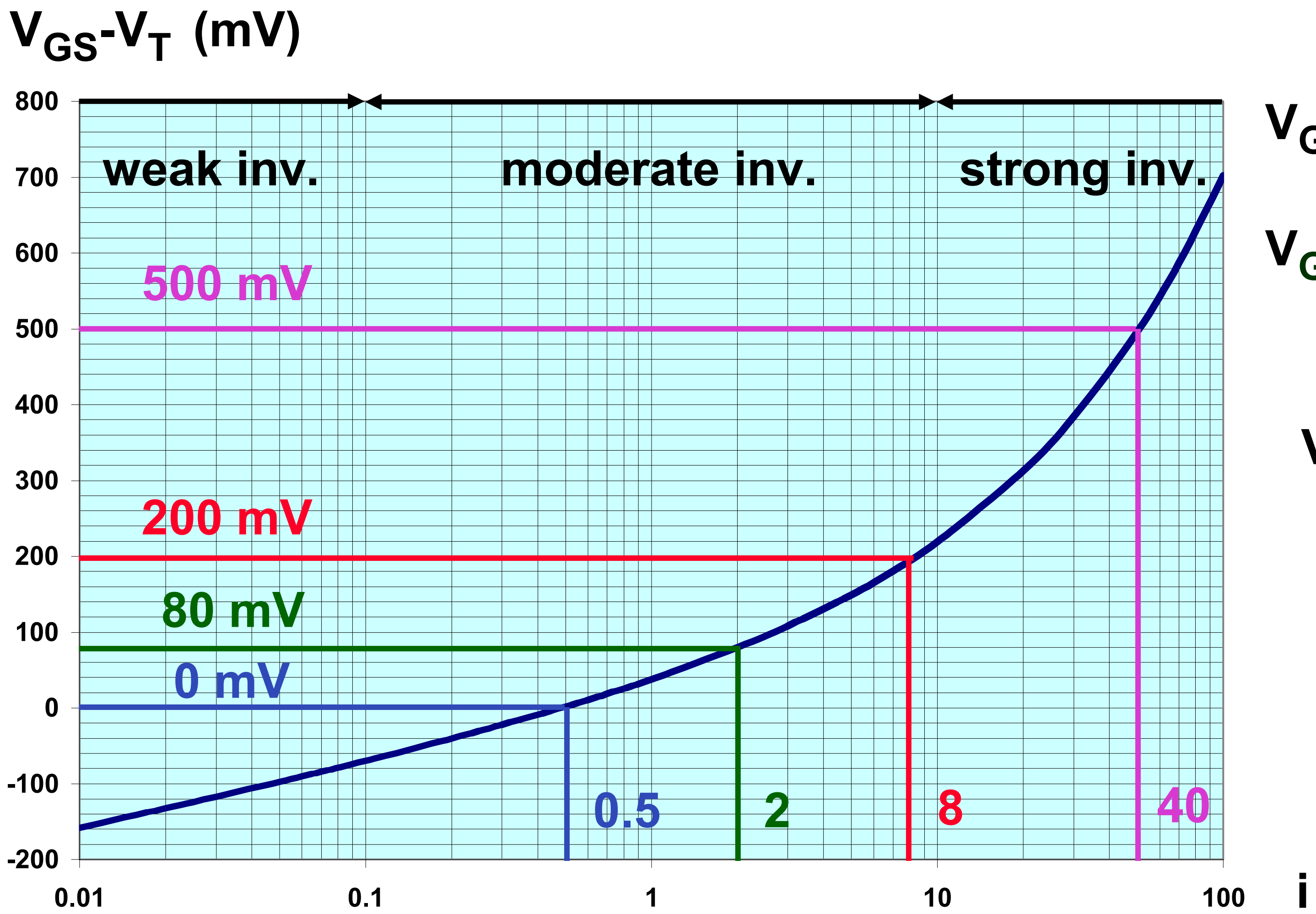
inversion coefficient

$$v = \ln(e^{\sqrt{i}} - 1)$$

$$V_{GS} - V_T = V_{GSTt} \ln(e^{\sqrt{i}} - 1)$$

$$V_{GSTt} = 2n \frac{kT}{q} \approx 70\ mV$$

Relation V_{GS}-V_T and inversion coefficient i



$V_{GS}-V_T =$

$V_{GSTt} \ln (e^{\sqrt{i}} - 1)$

$$V_{GSTt} = 2n \frac{kT}{q}$$

$$i = \frac{I_{DS}}{I_{DSt}}$$

Transconductance g_m between wi & si

$$i = \frac{I_{DS}}{I_{DSt}} = [\ \ln(1 + e^{v})\ ]^2 \qquad g_m \approx ....$$

$$GM = \frac{g_m}{I_{DS}} \frac{nkT}{q} = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

Large i : $GM = \dfrac{1}{\sqrt{i}}$

Small i : $GM = 1 - \dfrac{\sqrt{i}}{2}$

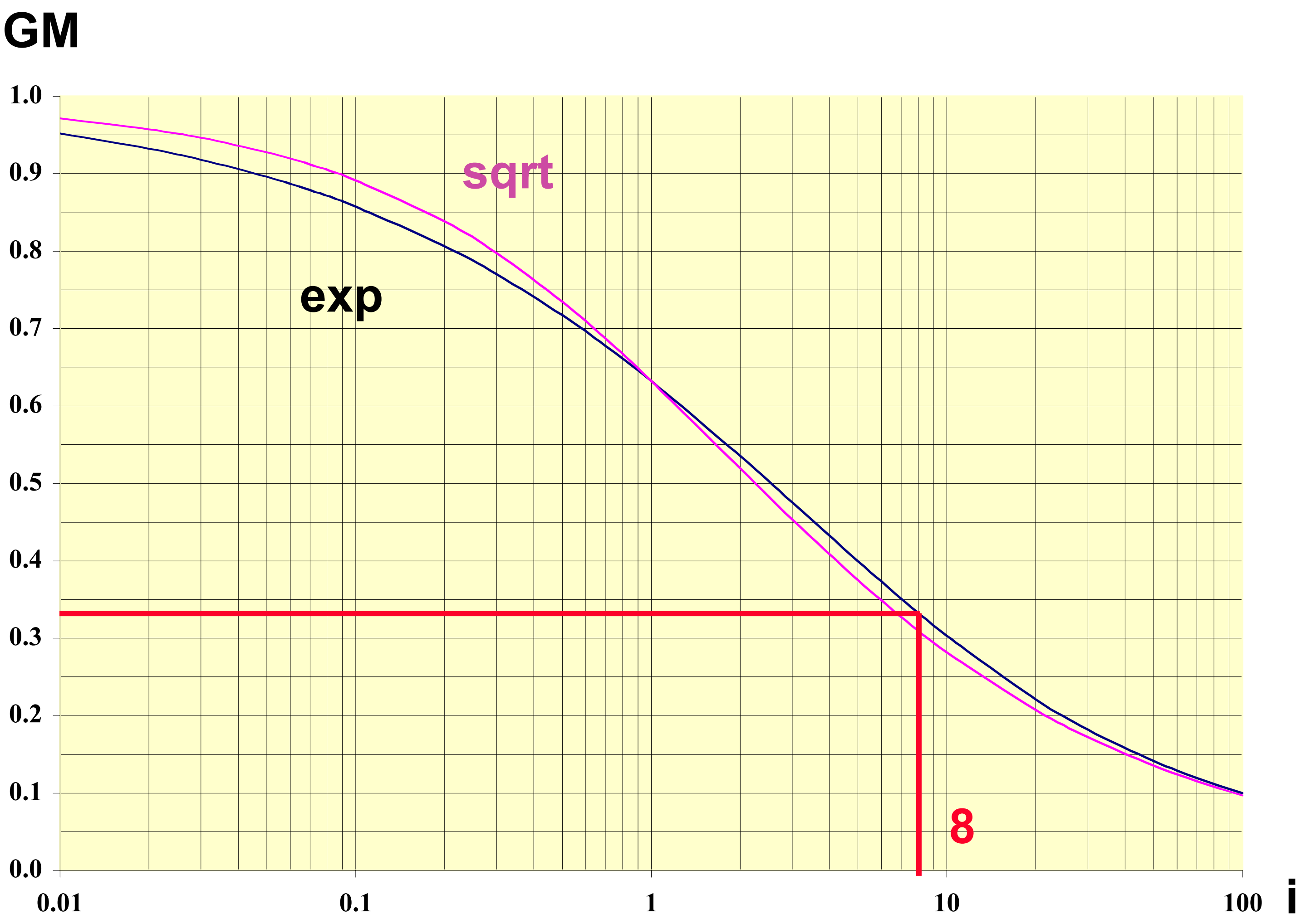
Alternative approximation :

$$GM = \frac{1}{\sqrt{1 + 0.5\sqrt{i} + i}}$$

Large i : $GM = \dfrac{1}{\sqrt{i}}$

Small i : $GM = 1 - \dfrac{\sqrt{i}}{4}$

GM versus inversion coefficient i



$$GM = \frac{g_m}{I_{DS}} \frac{nkT}{q}$$

$$GM = $$

$$\frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

$$\frac{1}{\sqrt{1 + 0.5\sqrt{i} + i}}$$

$$i = \frac{I_{DS}}{I_{DSt}}$$

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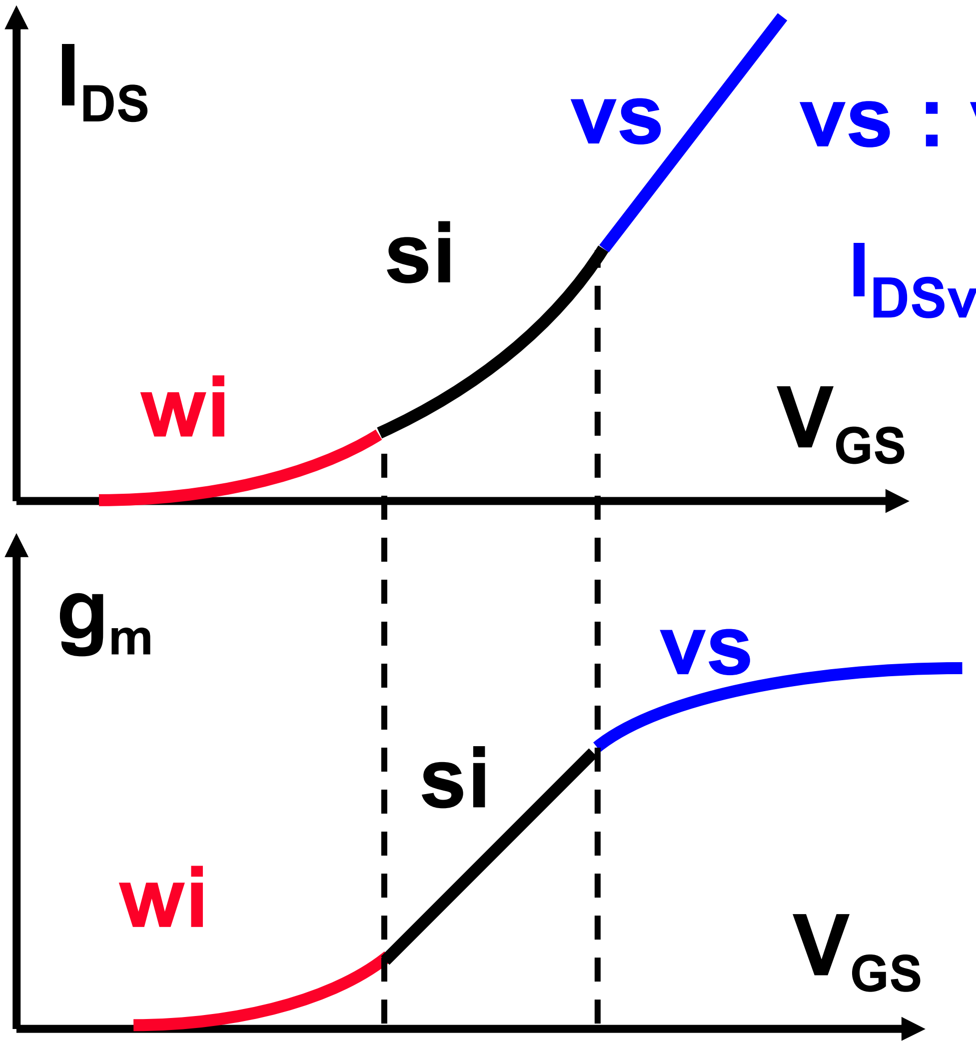
I_{DS} & g_m vs V_{GS} : velocity saturation



vs : velocity saturation

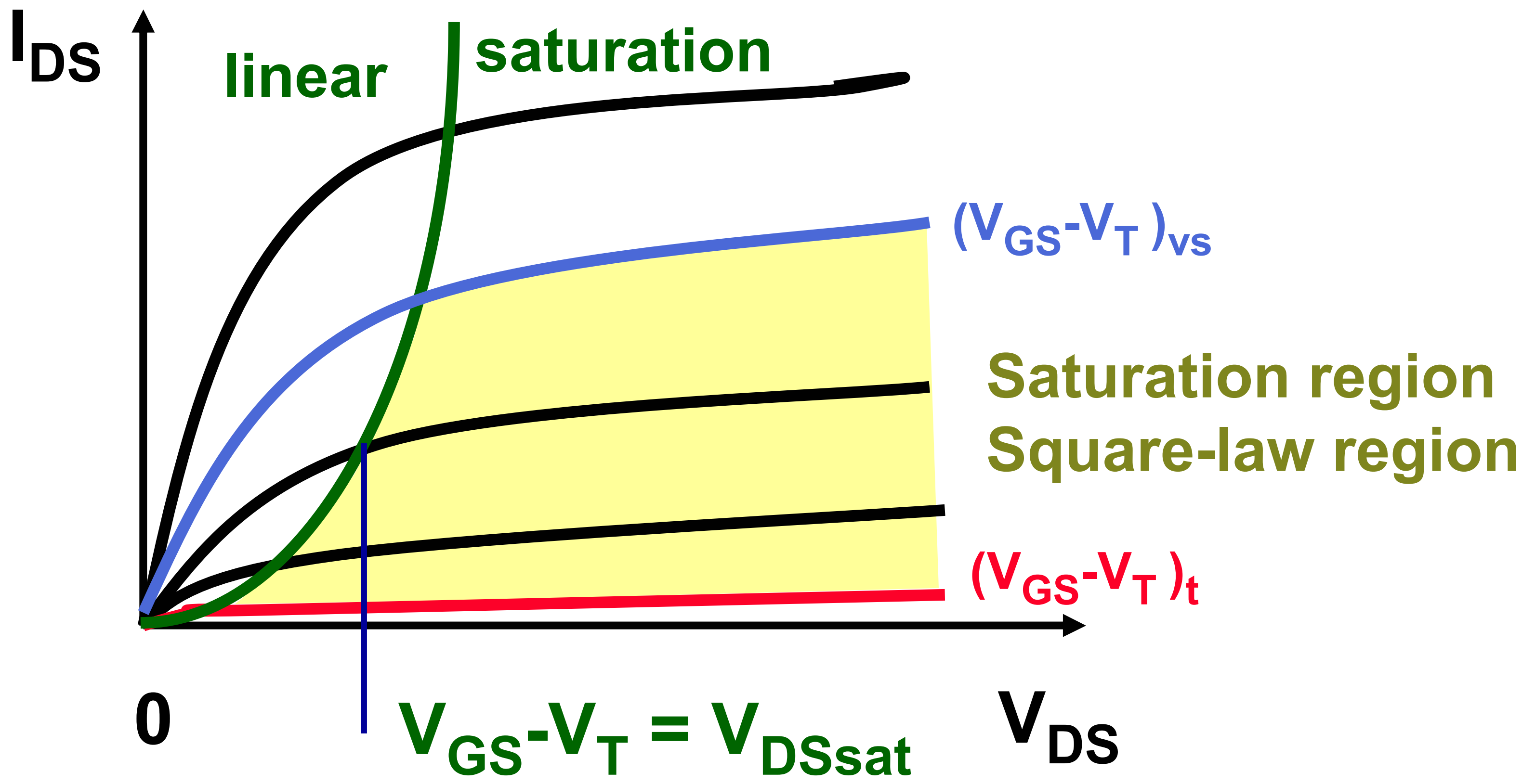
$$I_{DSvs} = WC_{ox}v_{sat}\,(V_{GS}-V_T)$$

$$v_{sat} \approx 10^7 \text{ cm/s}$$

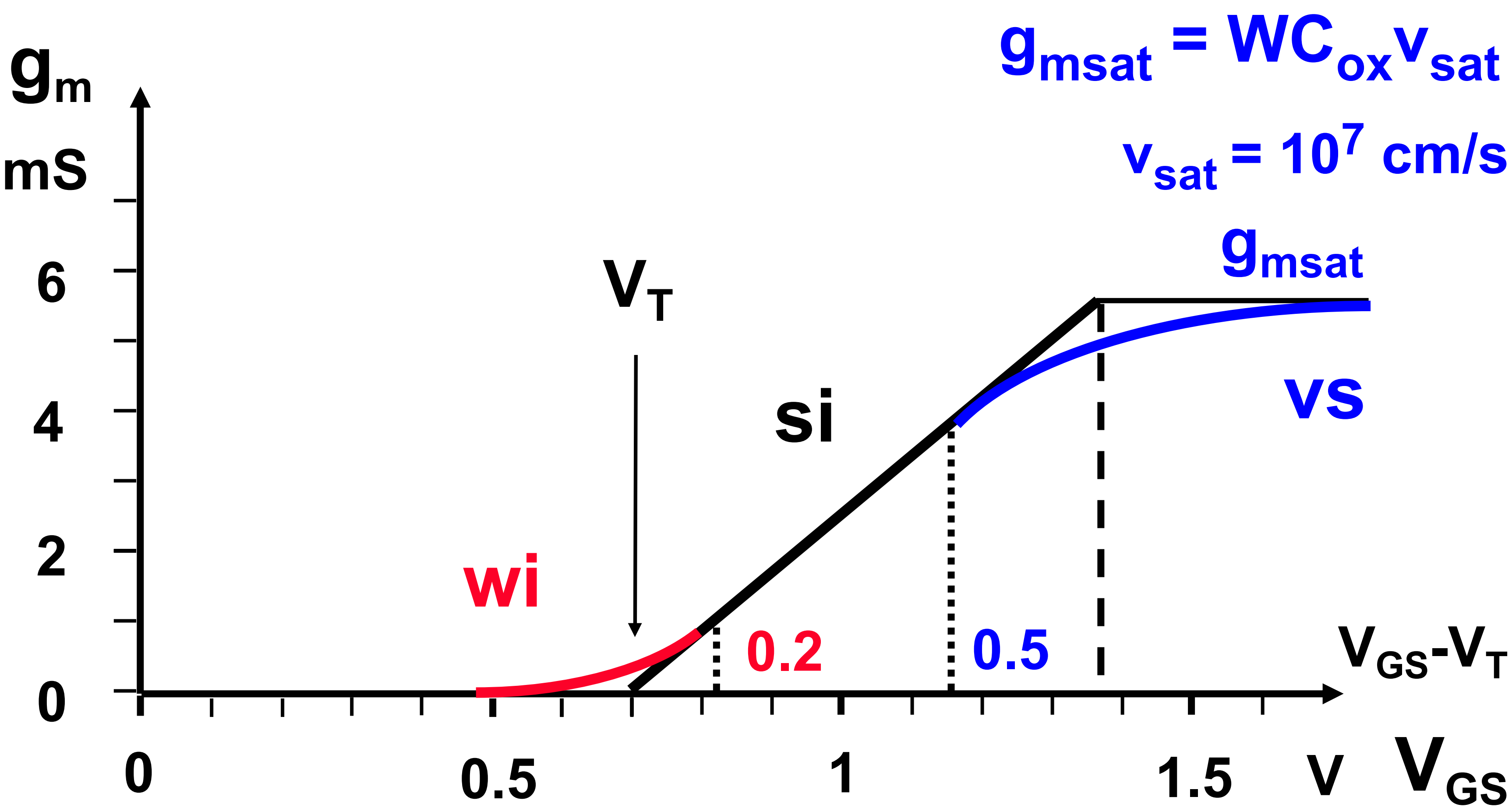
$$g_{msat} = WC_{ox}v_{sat}$$

is absolute max. !

The saturation region and velocity saturation



Transconductance g_m versus V_{GS}



Velocity saturation : v_{sat} & θ

[large V_{GS}]

$$I_{DS} = \frac{K'_n \frac{W}{L} (V_{GS}-V_T)^2}{1 + \theta (V_{GS}-V_T)} \approx \frac{K'_n}{\theta} \frac{W}{L} (V_{GS}-V_T)$$

$$g_{msat} \approx 2K'_n \frac{W}{L} (V_{GS}-V_T)^2 \frac{1 + \frac{\theta}{2}(V_{GS}-V_T)}{[1 + \theta (V_{GS}-V_T)]^2} \approx \frac{K'_n}{\theta} \frac{W}{L}$$

$= WC_{ox}v_{sat}$

$$\boxed{\theta L = \frac{\mu}{2n} \frac{1}{v_{sat}}} = \frac{1}{E_c}$$

E_c is the vertical critical field !

$\theta L \approx 0.2$ µm/V : For L = 0.13 µm $\theta \approx 1.6\ V^{-1}$

Velocity saturation : θ & R_S & v_{sat}

$$I_{DS} = \frac{K'_n \frac{W}{L} (V_{GS}-V_T)^2}{1 + \theta (V_{GS}-V_T)}$$

[large V_{GS}]

$$g_{msat} \approx \frac{K'_n}{\theta} \frac{W}{L}$$

$$g_{mRs} = \frac{g_m}{1 + g_m R_S} \approx \frac{1}{R_S}$$

$$R_S = \frac{\theta}{K'_n W/L}$$

$$R_S \approx \frac{\mu}{2n} \frac{1}{W K'_n v_{sat}} \approx \frac{1}{W C_{ox} v_{sat}}$$

Transition Voltage V_{GSTvs} between si and vs

$$I_{DS} = \frac{K'_n \frac{W}{L} (V_{GS}-V_T)^2}{1 + \theta (V_{GS}-V_T)}$$

$$I_{DSsat} = WC_{ox}v_{sat} (V_{GS}-V_T)$$

$$g_{msat} = WC_{ox}v_{sat} \approx \frac{K'_n}{\theta} \frac{W}{L}$$

$$(V_{GS}-V_T)_{vs} = \frac{1}{\theta} \approx 2nL \frac{v_{sat}}{\mu}$$

Is proportional to channel length L !!!

$\approx 5\,L \approx 0.62$ V if L = 0.13 μm

Transition Current I_{DSvs} between si and vs

$$I_{DSvs} \approx K' WL \left(\frac{2n\, v_{sat}}{\mu}\right)^2 \approx 100\, n\, \varepsilon_{ox} W \frac{v_{sat}^2}{\mu}$$

$$\frac{I_{DSvs}}{W} \approx 10 \text{ A/cm}$$

$$K' = \frac{\mu C_{ox}}{2n}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \qquad t_{ox} = \frac{L}{50}$$

W = 2.6 μm & L = 0.13 μm :

$I_{DSvs} \approx 2.6$ mA

$v_{sat} = 10^7$ cm/s

n = 1.4

μ = 500 cm^2/Vs

Transconductance g_m between si and vs

$g_{msat} = W\,C_{ox}\,v_{sat}$

$g_{msat} \approx 17\ 10^{-5}$ W/L S/cm

$$g_{mK'} = 2K'\,\frac{W}{L}\,\underbrace{(V_{GS}-V_T)}_{V_{GST}}$$

$g_{mK'} \approx 1.2\ 10^{-9}\ V_{GST}$ W/L^2 S/cm

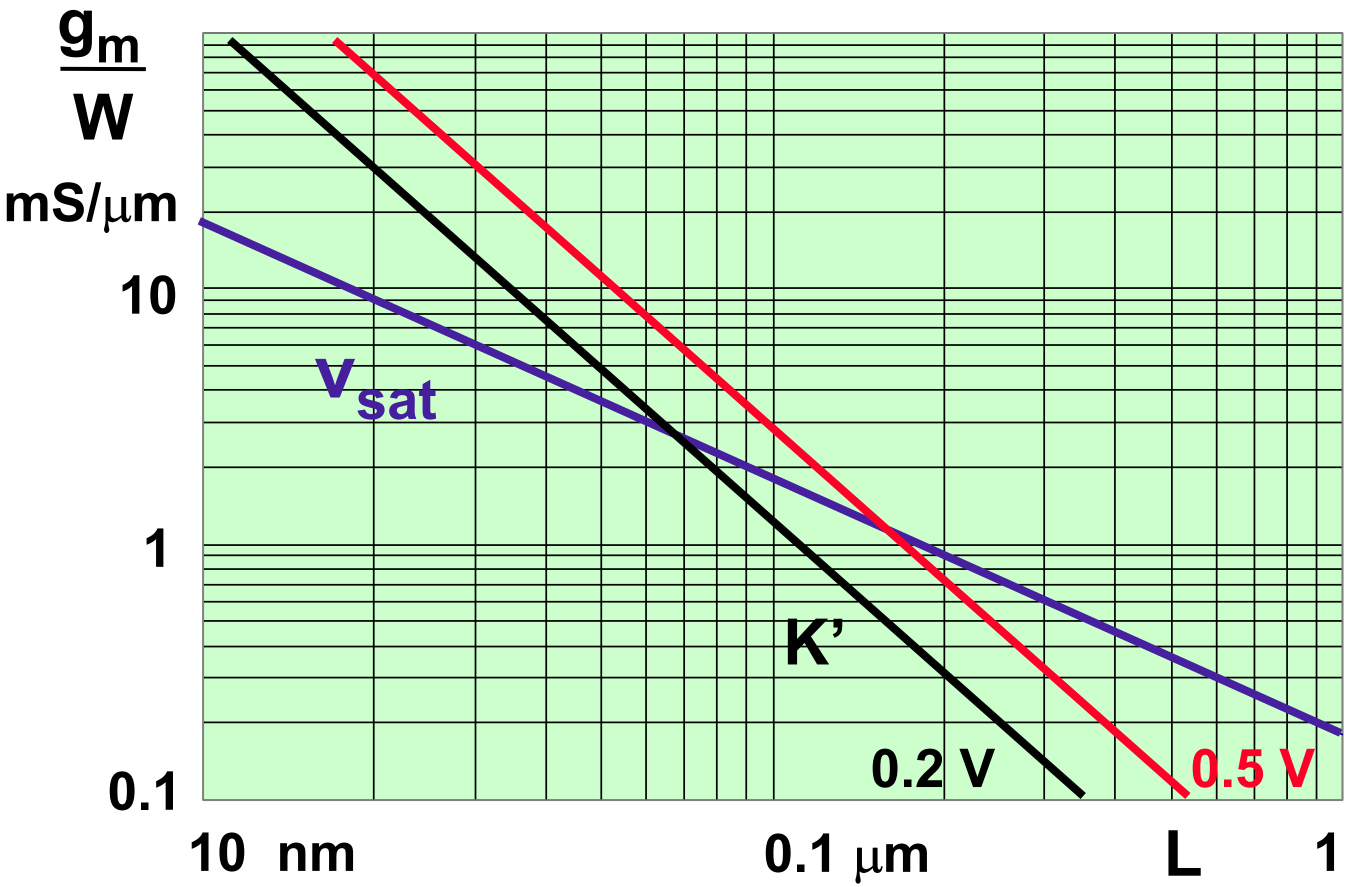
$$\frac{1}{g_m} = \frac{1}{g_{mK'}} + \frac{1}{g_{msat}}$$

$$g_m \approx \frac{W}{L}\,\frac{17\ 10^{-5}}{1 + 2.8\ 10^4\ L / V_{GST}}$$

L in cm

If V_{GST} = 0.2 V, v_{sat} takes over for L < 65 nm (If 0.5 V for L < 0.15 μm)

Now in velocity saturation ?



$V_{GS}-V_T \approx$ 0.2 V
(0.5 V)

$$\frac{g_m}{W} = \frac{2K'}{L}(V_{GS}-V_T)$$

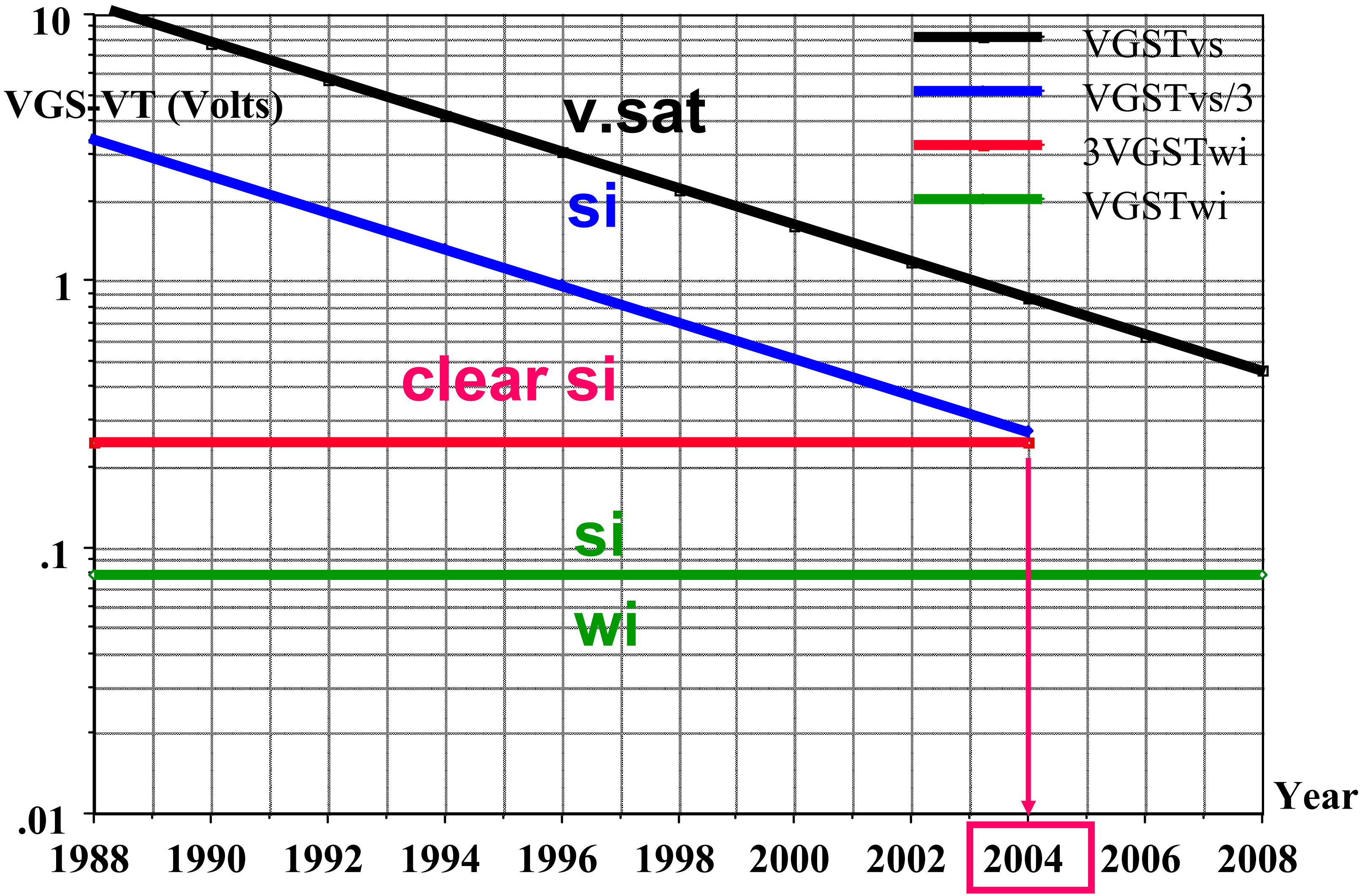
$$\frac{g_m}{W} \approx \frac{0.06\, V_{GST}}{L^2}$$ in μm

$$\frac{g_m}{W} = C_{ox}\, v_{sat}$$

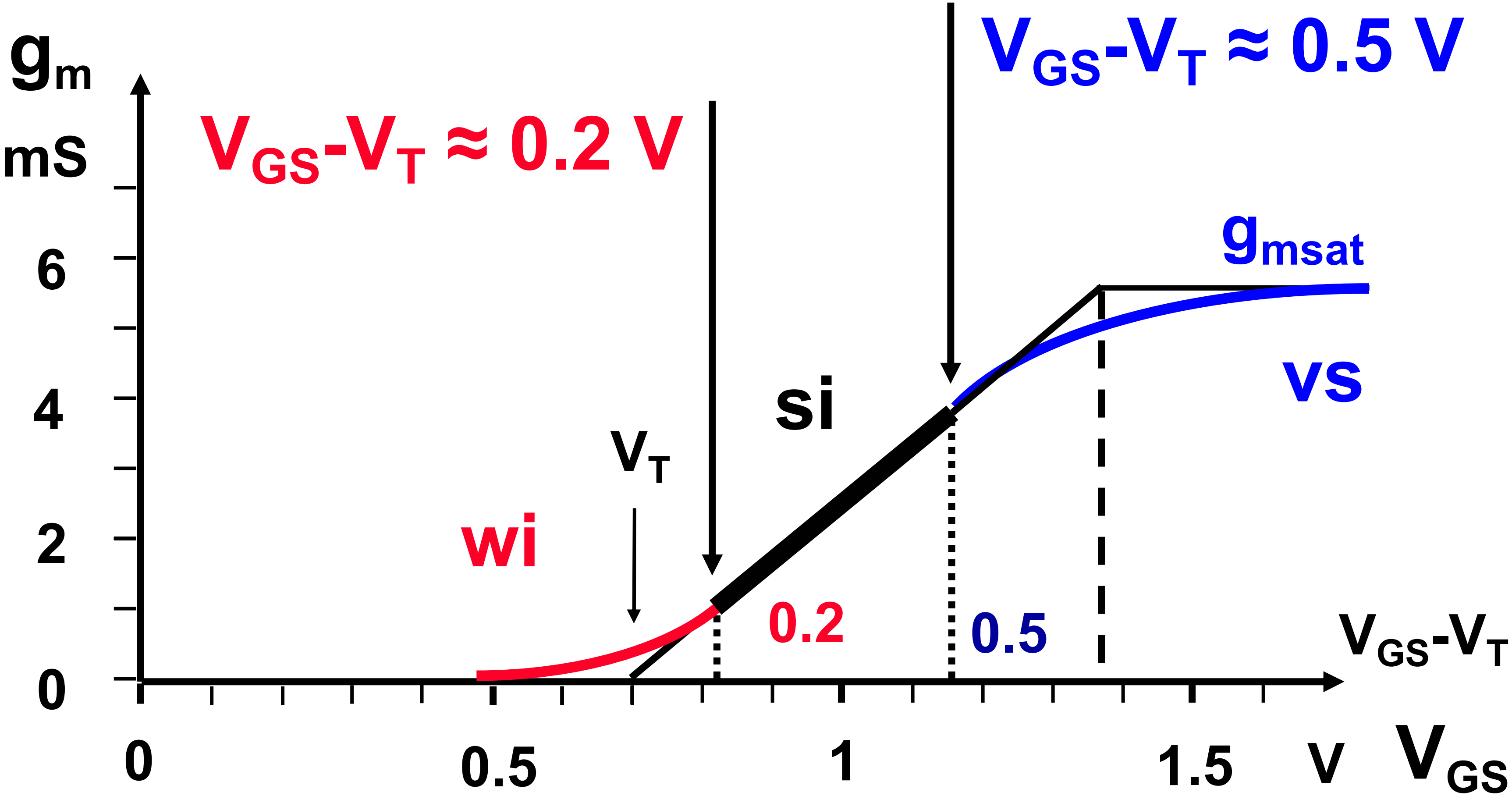
$$\frac{g_m}{W} \approx \frac{0.17}{L}$$

in μm

Range of V_{GS}-V_T values for si vs time

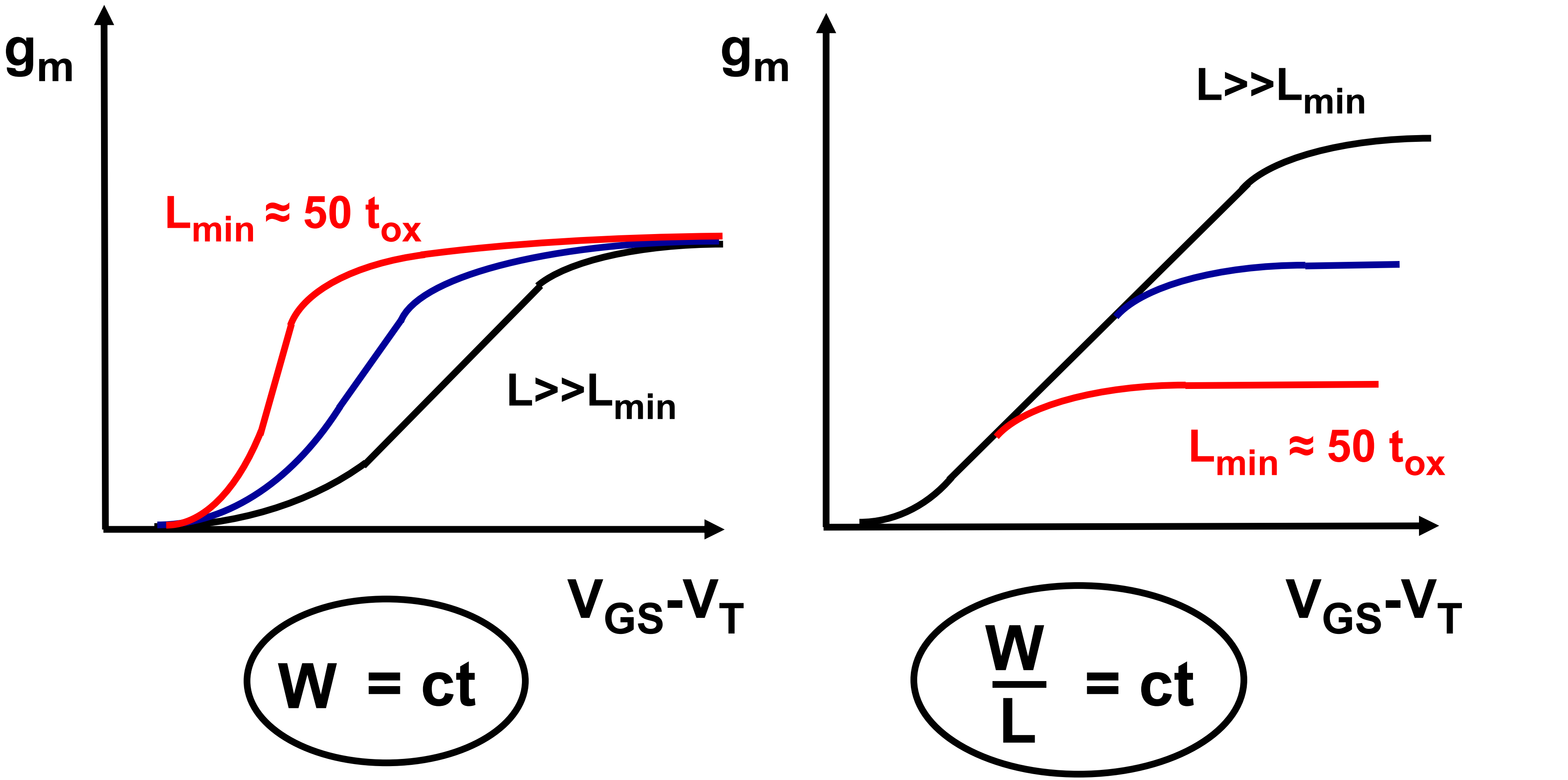


MOST operating region in si



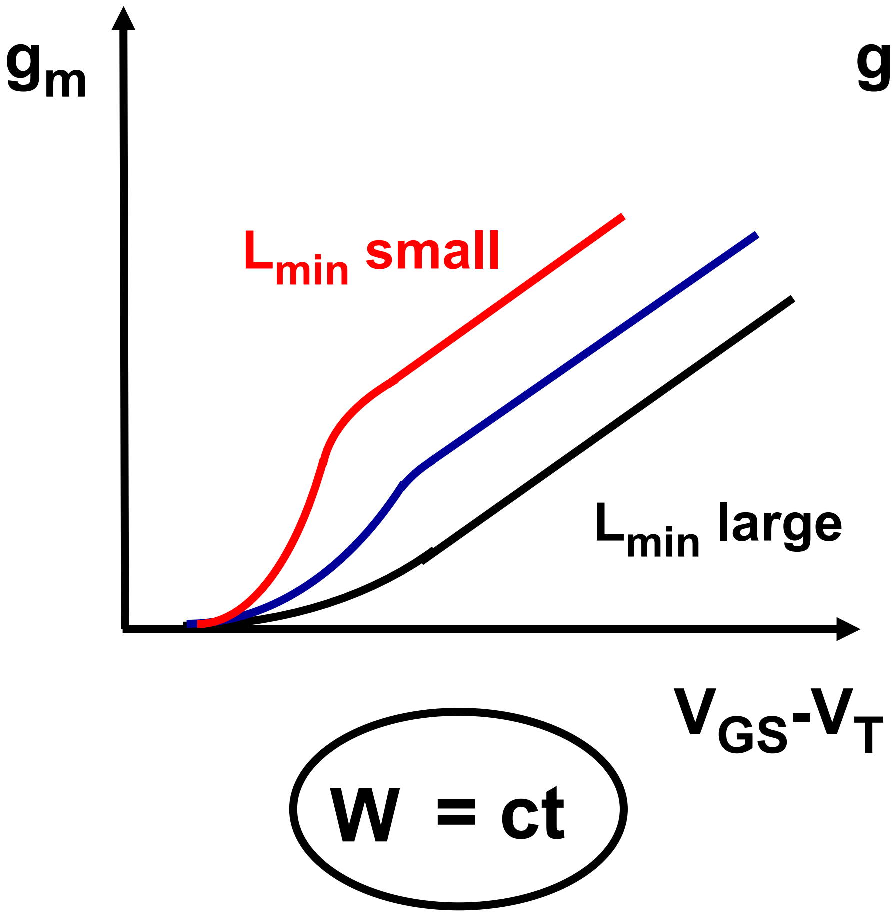
g_m vs V_{GS} for different L (same t_{ox})

Exercise :



g_m vs V_{GS} for different t_{ox} ($\approx L_{min}/50$)

Exercise :



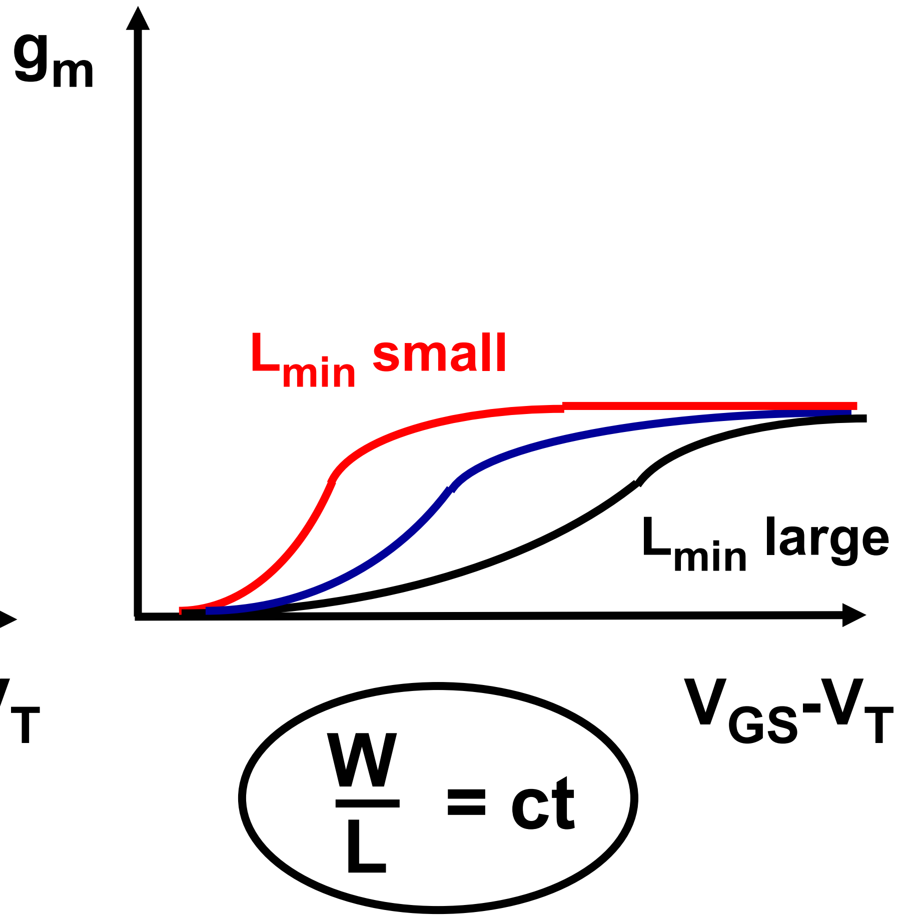


Table : MOST I_{DS} , g_m & g_m/I_{DS}

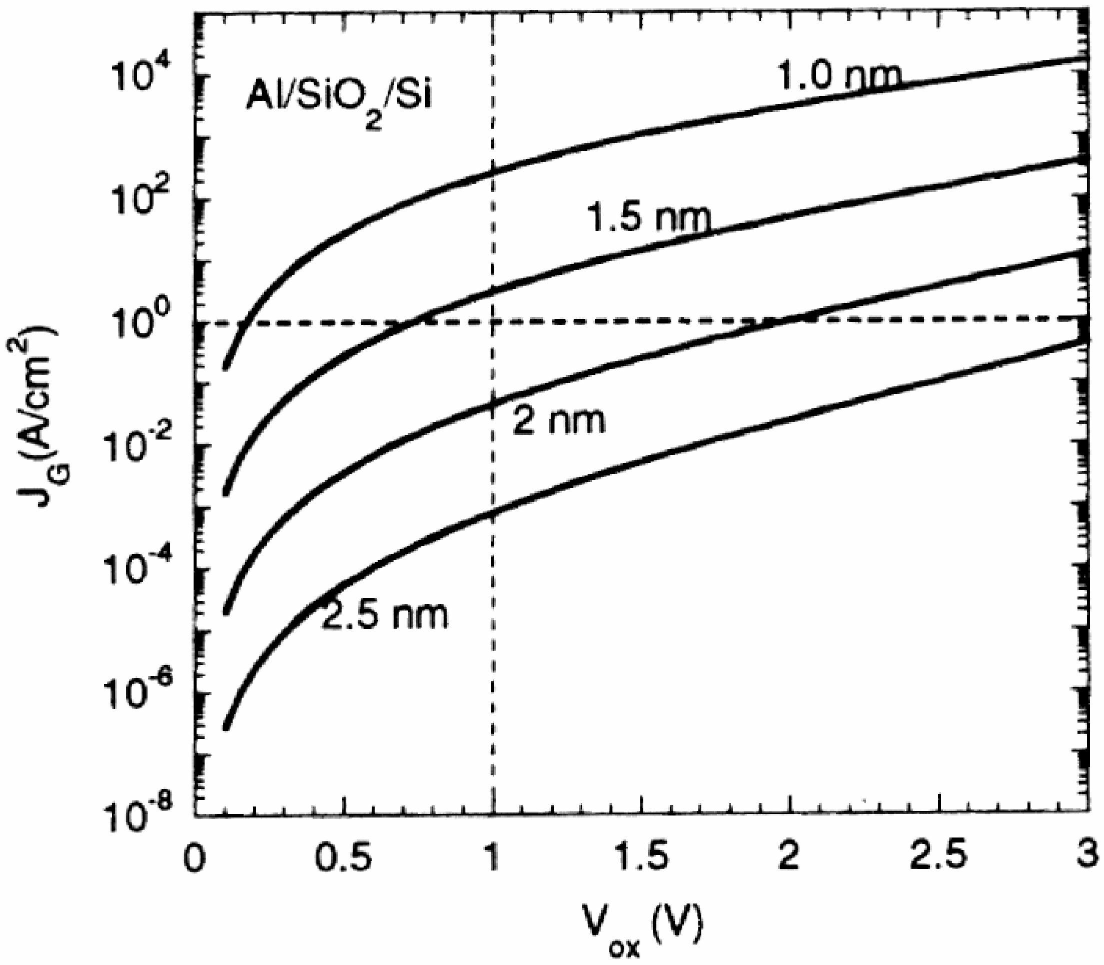
Summary :

TABLE 1-4 EXPRESSIONS OF I_{DS}, g_m AND g_m/I_{DS} FOR MOST

	I_{DS}	g_m	$\frac{g_m}{I_{DS}} = f(v_{GS} - V_T)$	$\frac{g_m}{I_{DS}} = f(I_{DS})$
wi	$I_{D0}\frac{W}{L}\exp\left(\frac{v_{GS}}{nkT/q}\right)$ (1-25*a*)	$\frac{I_{D0}}{nkT/q}\frac{W}{L}\exp\left(\frac{v_{GS}}{nkT/q}\right)$ (1-25*b*)	$\frac{1}{nkT/q}$ (1-26*b*)	$\frac{1}{nkT/q}$ (1-26*b*)
ws			$(v_{GS} - V_T)_{ws} = 2n\frac{kT}{q}$	$I_{DSws} = \frac{KP}{2n}\frac{W}{L}\left(2n\frac{kT}{q}\right)^2$
si	$\frac{KP}{2n}\frac{W}{L}(v_{GS} - V_T)^2$ (1-18*c*)	$2\frac{KP}{2n}\frac{W}{L}(v_{GS} - V_T)$ (1-22*a*)	$\frac{2}{v_{GS} - V_T}$ (1-26*a*)	$2\sqrt{\frac{KP}{2n}\frac{W}{L}\frac{1}{I_{DS}}}$ (1-26*a*)
sv			$(v_{GS} - V_T)_{sv} = \frac{2nLC_{ox}v_{sat}}{KP}$	$I_{DSsv} = \frac{2WLC_{ox}^2 v_{sat}^2}{KP/2n}$
vs	$WC_{ox}v_{sat}(v_{GS} - V_T)$ (1-38*b*)	$WC_{ox}v_{sat}$ (1-39)	$\frac{1}{v_{GS} - V_T}$	$\frac{WC_{ox}v_{sat}}{I_{DS}}$

Ref.: Laker, Sansen : Design of analog …, MacGrawHill 1994; Table 1-4

Gate current



For 0.1 μm CMOS :

$t_{ox} \approx 2$ nm

$J_G \approx 4\ 10^{-2}$ A/cm^2

For 10 x 0.5 μm

$I_G \approx 2$ nA

J_G (A/cm^2)

$\approx 4.5\ 10^5 \exp(-\frac{L}{6.5})$

L in nm

Ref. Koh, Tr ED 2001, 259-
Annema, JSSC Jan.05, 135.

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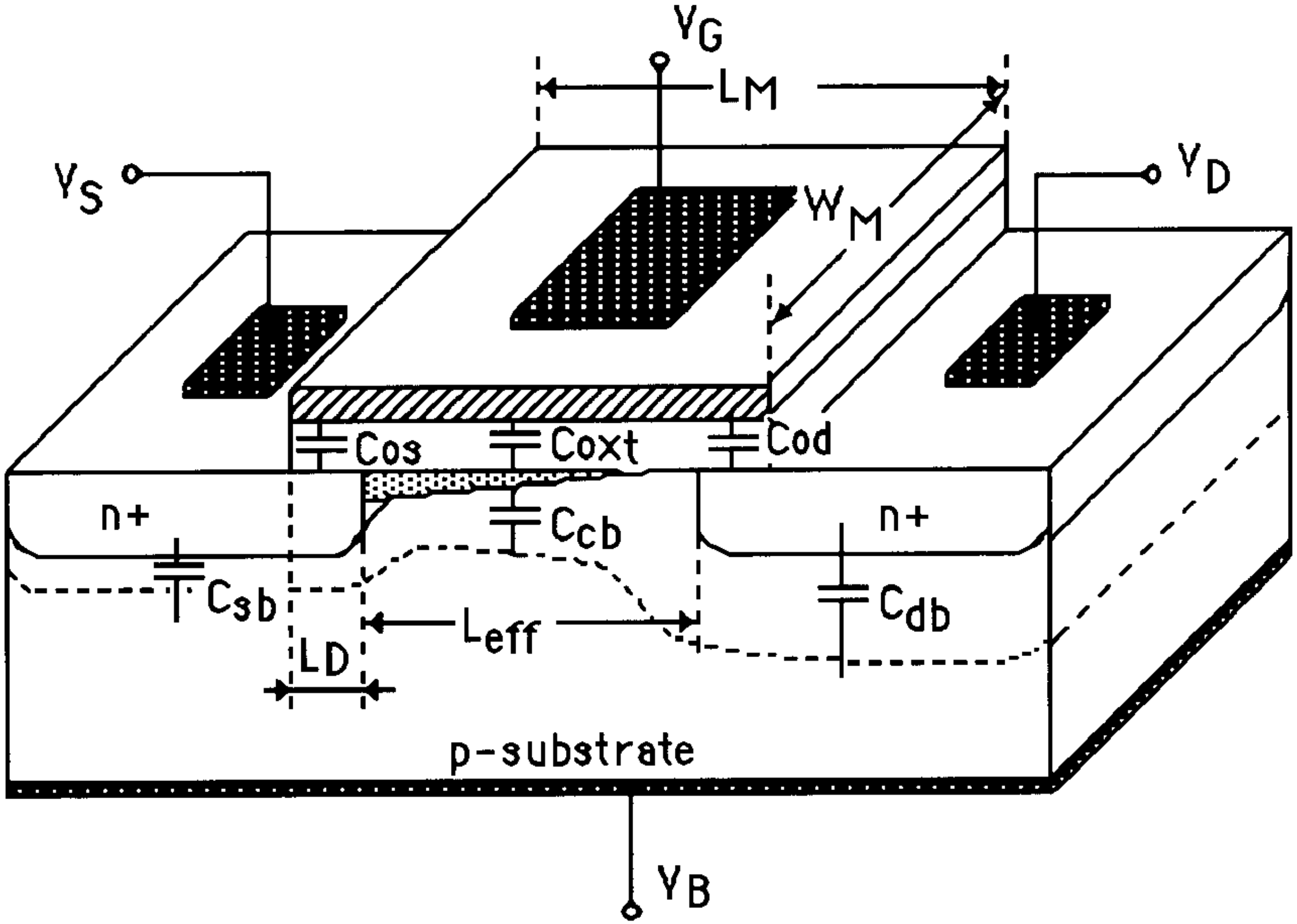
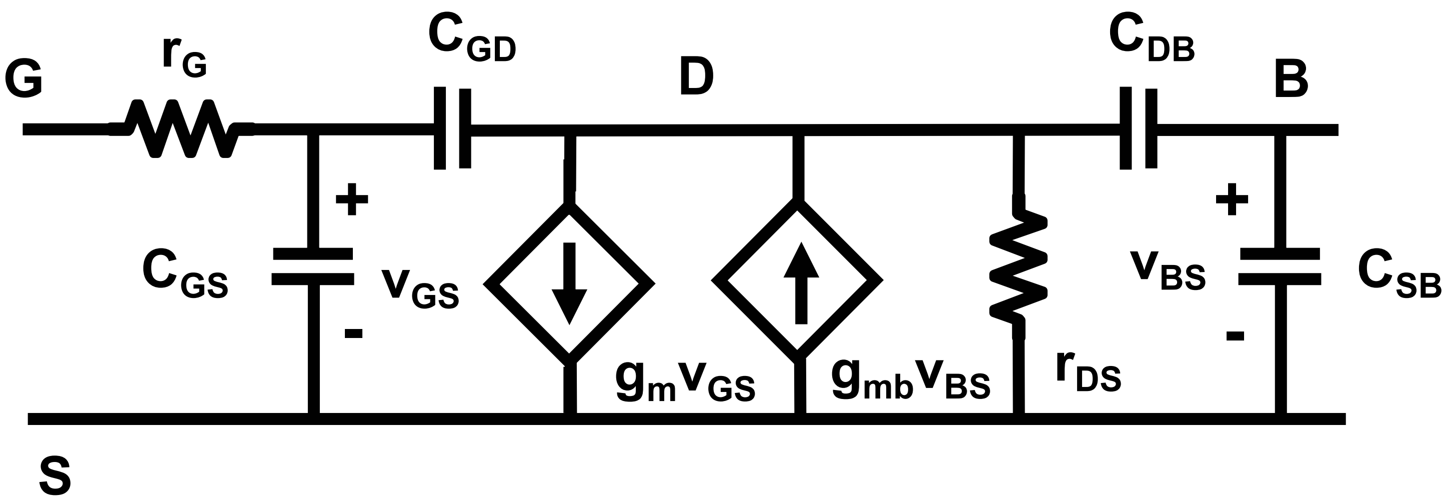
- **Models of MOST transistors**
  - **MOST as a resistor**
  - **MOST as an amplifier in strong inversion**
  - **Transition weak inversion-strong inversion**
  - **Transition strong inversion-velocity saturation**
  - **Capacitances and f_T**
- **Models of Bipolar transistors**
- **Comparison of MOSTs & Bipolar transistors**

MOST capacitances



MOST capacitances C_{GS} & C_{GD}

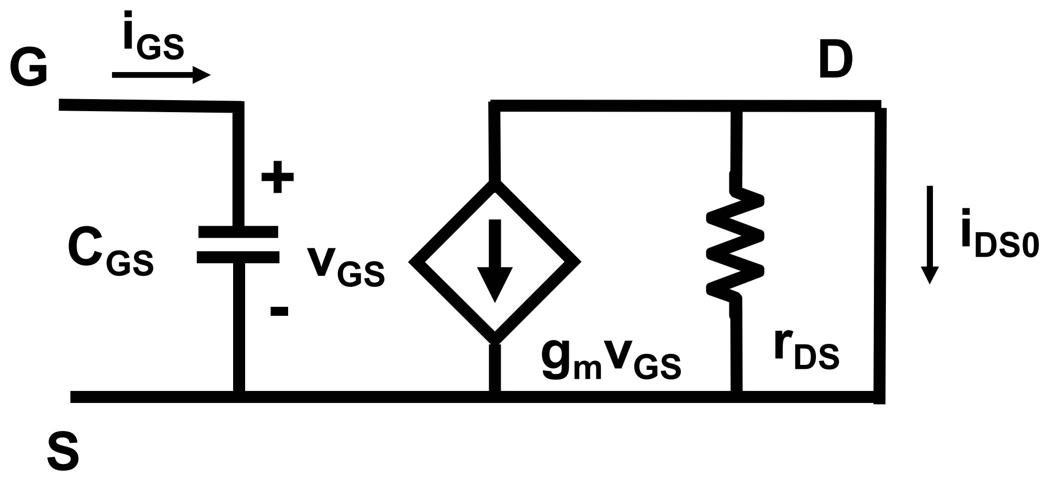


$$C_{GS} \approx \frac{2}{3} WLC_{ox} \quad \approx 2W \;\; \text{fF}/\mu\text{m for } L_{min}$$

$$L_{min}C_{ox} \approx L_{min} \frac{\varepsilon_{ox}}{t_{ox}} \approx 50\, \varepsilon_{ox} \approx 2 \text{ fF}/\mu\text{m}$$

$$C_{GD} = WC_{gdo}$$

MOST f_T where $i_{DS} = i_{GS}$



$$i_{GS} = v_{GS} C_{GS} s$$

$$i_{DS} = g_m v_{GS}$$

$$C_{GS} = \frac{2}{3} WLC_{ox} \quad g_m = 2K' \frac{W}{L} (V_{GS}-V_T) \quad K' = \frac{\mu C_{ox}}{2n}$$

$$f_T = \frac{g_m}{2\pi C_{GS}} = \frac{1}{2\pi} \frac{3}{2n} \frac{\mu}{L^2} (V_{GS}-V_T)$$

or $\approx \frac{v_{sat}}{2\pi L}$

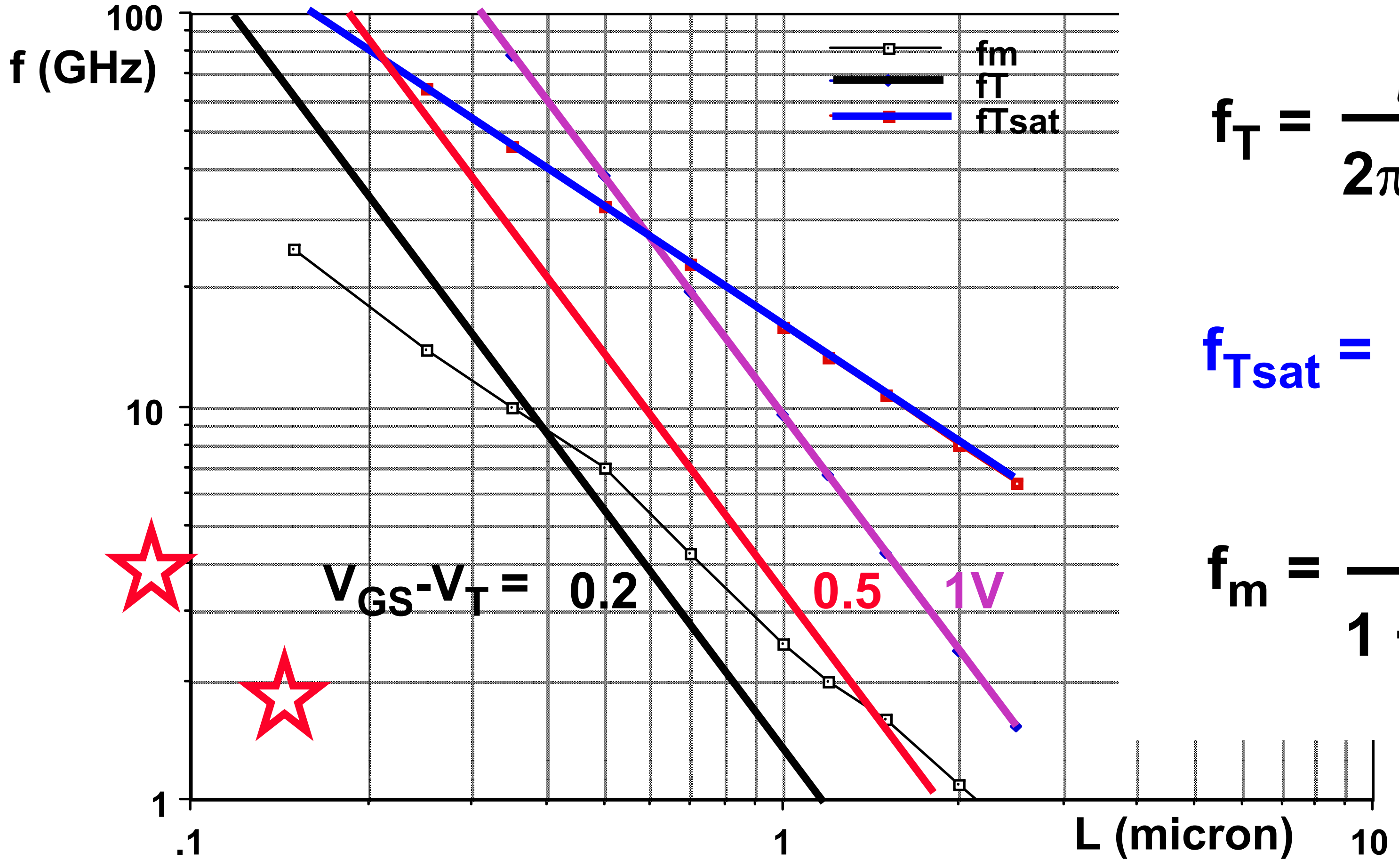
$$f_{max} \approx \sqrt{f_T / 8\pi r_G C_{GD}}$$

Design for high speed :

	High gain	High speed
V_{GS}-V_T	Low (0.2 V)	High (0.5 V)
L	High	Low

V_{GS}-V_T sets the ratio g_m/I_{DS} !

Maximum f_T values versus channel length L



Processors

$$f_T = \frac{\mu}{2\pi L^2} \underbrace{(V_{GS}-V_T)}_{0.2\ \ldots\ 1\ V}$$

$$f_{Tsat} = \frac{v_{sat}}{2\pi L}$$

$$f_m = \frac{f_T}{1 + \alpha_{BD}}$$

$$\alpha_{BD} \approx \frac{C_{BD}}{C_{ox}}$$

f_T model in si and velocity saturation

$$f_T = \frac{g_m}{2\pi C_{GS}}$$

$C_{GS} = kW$ $\quad k = 2\ fF/\mu m = 2\ 10^{-11}\ F/cm$

$$g_m = \frac{W}{L} \frac{17\ 10^{-5}}{1 + 2.8\ 10^{4}\ L / V_{GST}}$$ L in cm

$$f_T = \frac{1}{L} \frac{13.5}{1 + 2.8\ L / V_{GST}}\ \text{GHz}$$ L in μm

If $V_{GST} = 0.2$ V, v_{sat} takes over for L < 65 nm

If $V_{GST} = 0.5$ V for L < 0.15 μm

f_T model in si and weak inversion

$$f_T = \frac{g_m}{2\pi\, C_{GS}}$$

$$GM = \frac{g_m}{I_{DS}}\,\frac{nkT}{q} = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

$$g_m = \frac{I_{DS}}{nkT/q}\,\frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$ but $I_{DS} = i\, I_{DSt}$

$$g_m = \frac{I_{DSt}}{nkT/q}\,\sqrt{i}\,(1 - e^{-\sqrt{i}})$$

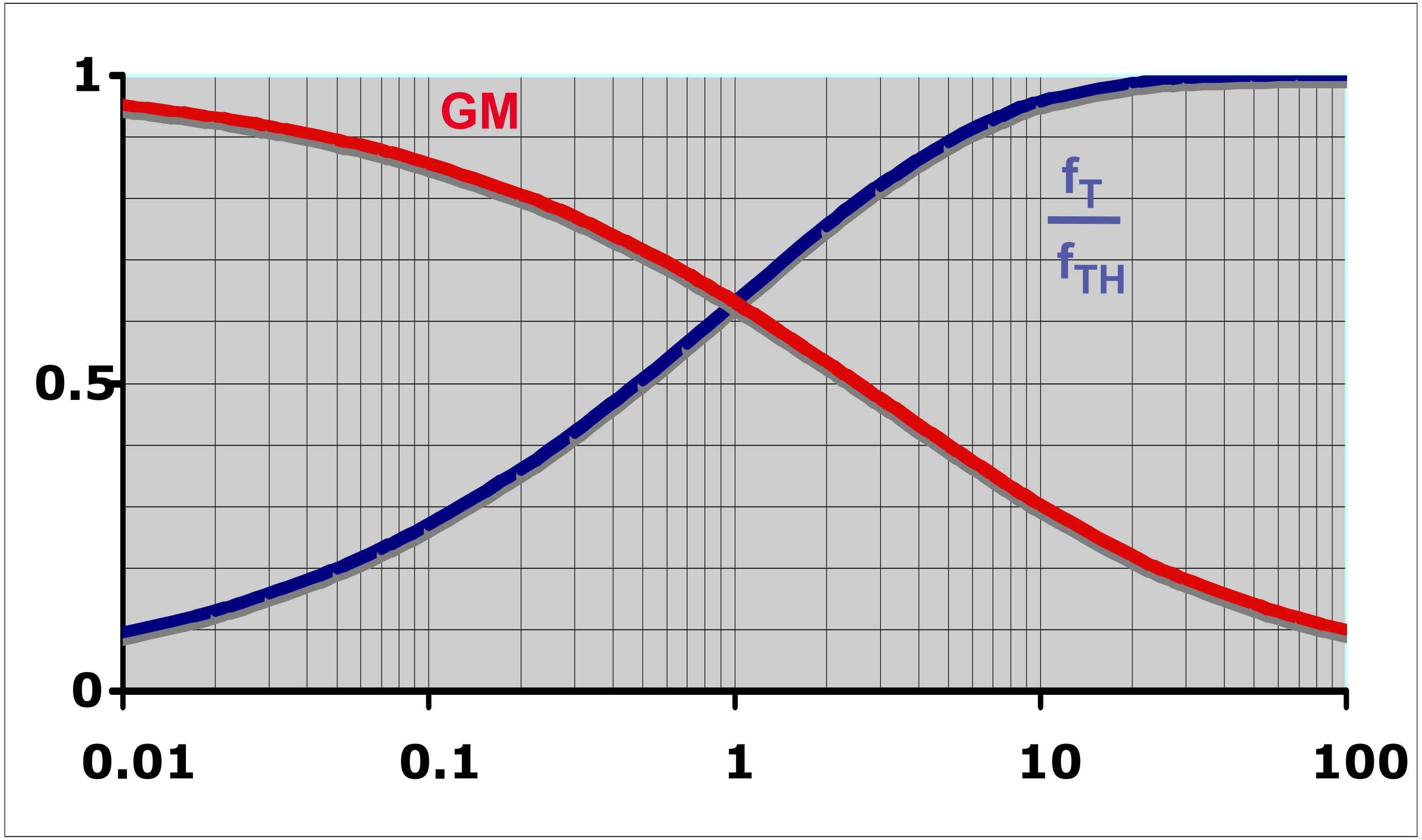
$$\frac{f_T}{f_{TH}} = \sqrt{i}\,(1 - e^{-\sqrt{i}})$$

$\approx$ i for small i !

$$f_{TH} = \frac{I_{DSt}}{2\pi\, C_{GS}\, nkT/q} = \frac{K'\, V_{GSTt}^{2}\, W/L}{2\pi\, WL\, C_{ox}\, nkT/q}$$

$$= \frac{4\,K'\, nkT/q}{2\pi\, C_{ox}\, L^2} = \frac{2\,\mu\, kT/q}{2\pi\, L^2}$$

f_T versus inversion coefficient i



$$\frac{f_T}{f_{TH}} = \sqrt{i}\,(1 - e^{-\sqrt{i}})$$

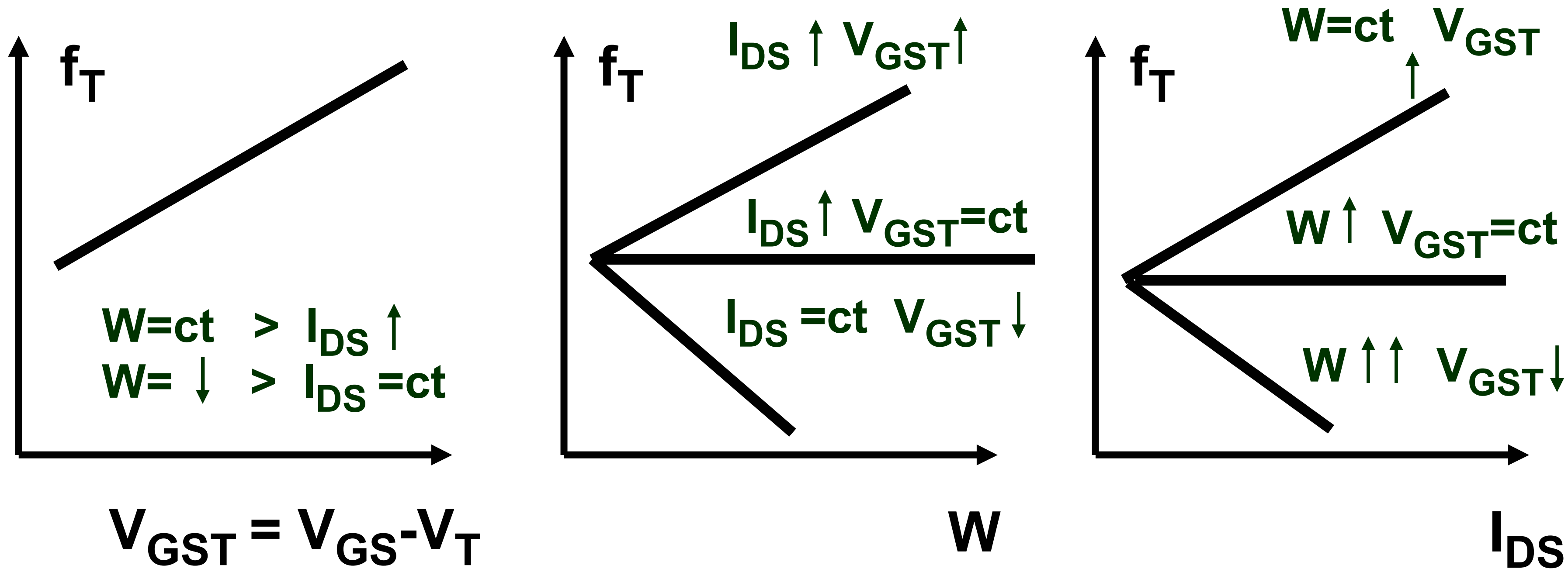
$$GM = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

$$i = \frac{I_{DS}}{I_{DSt}}$$

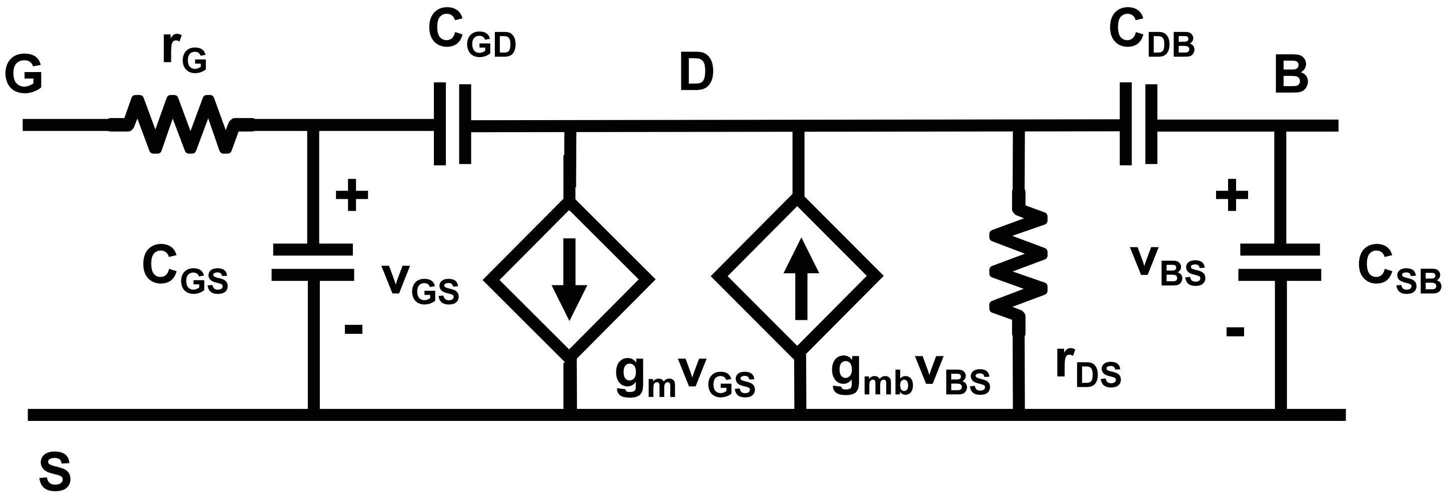
Exercise: MOST f_T or not f_T ?

all $L = L_{min}$

$$f_T = \frac{1}{2\pi}\frac{\mu}{L^2}(V_{GS}-V_T) = \frac{\sqrt{K'I_{DS}}}{\pi C_{ox}\sqrt{WL^3}} = \frac{I_{DS}}{\pi WLC_{ox}(V_{GS}-V_T)}$$



MOST capacitances C_{SB} & C_{DB}

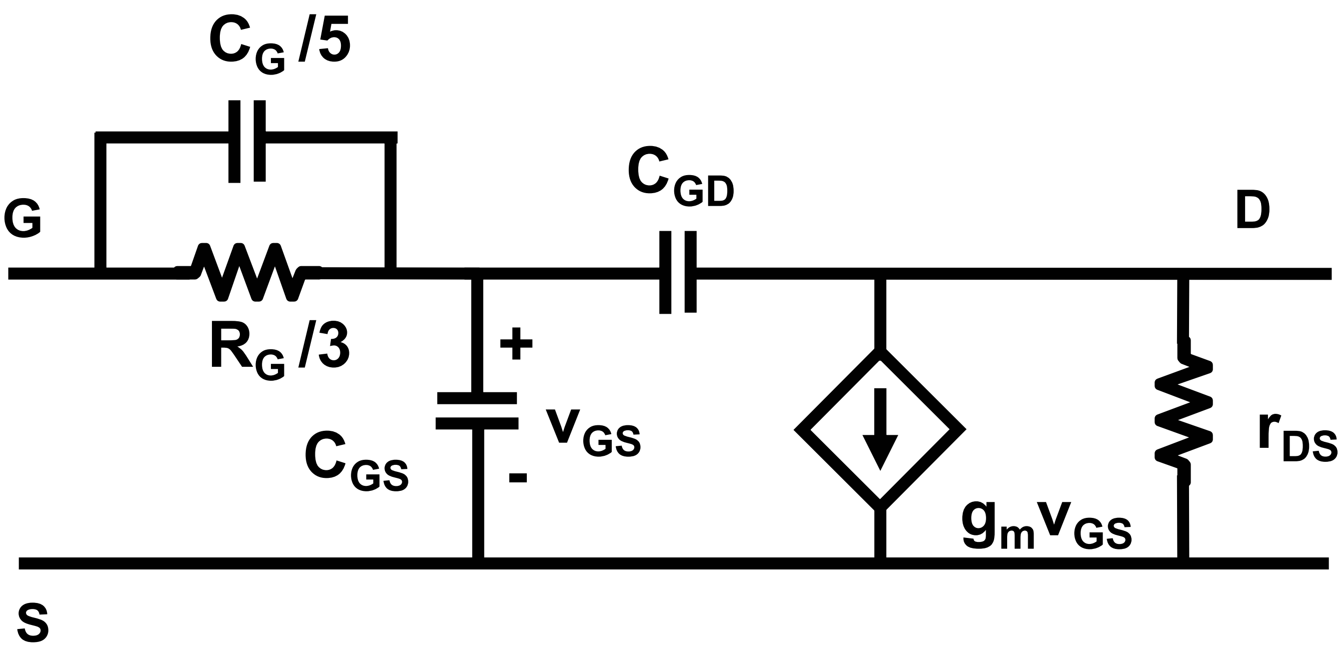


$$C_{SB} = \frac{C_{jSB0}}{\sqrt{1 + V_{SB}/\phi_{jS}}}$$

$$C_{DB} = \frac{C_{jDB0}}{\sqrt{1 + V_{DB}/\phi_{jD}}}$$

$\phi_{jS} \approx \phi_{jD} \approx 0.5 \dots 0.7$ V

RF MOST model



$$C_G = C_{GS} + C_{GD}$$

Ref. Tin, Tr. CAD, April 1998, 372

Ref. Sansen, etal, ACD, XDSL, RFMOS models, Kluwer 1999

Single-page MOST model

$$I_{DS} = K'_n \frac{W}{L} (V_{GS}-V_T)^2$$

$V_{GS}-V_T \approx 0.2$ V

$K'_n \approx 100\ \mu A/V^2$

$K'_p \approx 40\ \mu A/V^2$

$$g_m = 2K'_n \frac{W}{L} (V_{GS}-V_T) = 2\sqrt{K'_n \frac{W}{L} I_{DS}} = \frac{2\, I_{DS}}{V_{GS}-V_T}$$

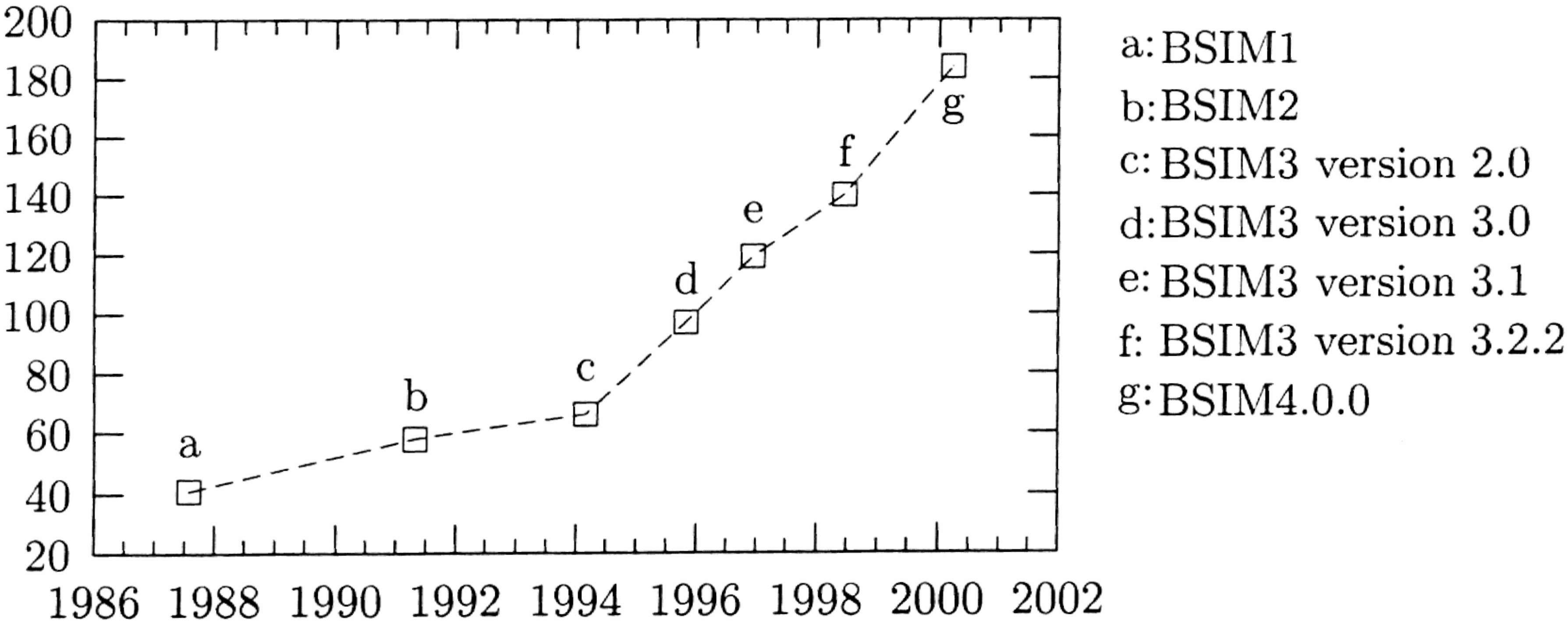
$$r_{DS} = r_o = \frac{V_E L}{I_{DS}}$$

$V_{En} \approx 5$ V/µmL $V_{Ep} \approx 8$ V/µmL

$v_{sat} = 10^7$ cm/s

$$f_T = \frac{1}{2\pi} \frac{3}{2n} \frac{\mu}{L^2} (V_{GS}-V_T) \quad \text{or now} \approx \frac{v_{sat}}{2\pi L}$$

Growing number of parameters !



BSIM4 : http://www-device.eecs.berkeley.edu/bsim/bsim_ent.html

Model 11 : http://www.semiconductors.philips.com/Philips_Models/mos_models/model11/index.html

EKV : http://legwww.epfl.ch/ekv/model.html

Benchmark tests

1. Weak inversion transition for I_{DS} and g_m/I_{DS} ratio
2. Velocity saturation transition for I_{DS} and g_m/I_{DS} ratio
3. Output conductance around V_{DSsat}
4. Continuity of currents and caps around zero V_{DS}
5. Thermal and 1/f noise
6. High frequency input impedance (s_{11}) and
   transimpedance (s_{21})

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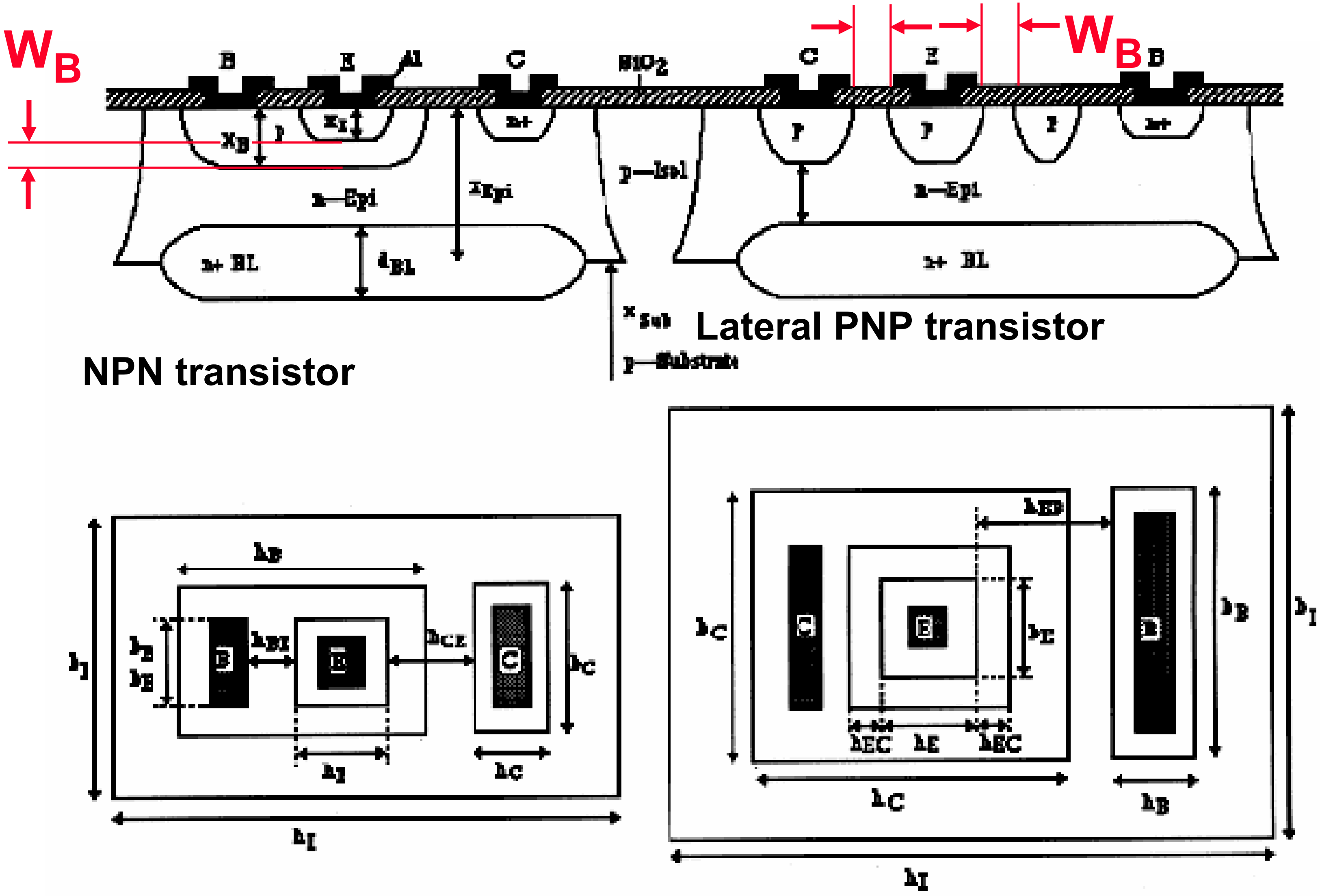
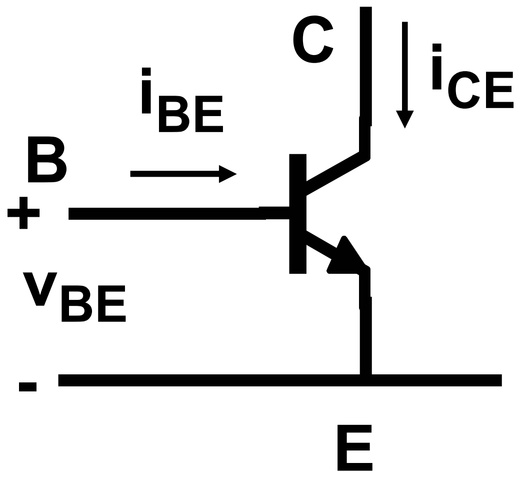
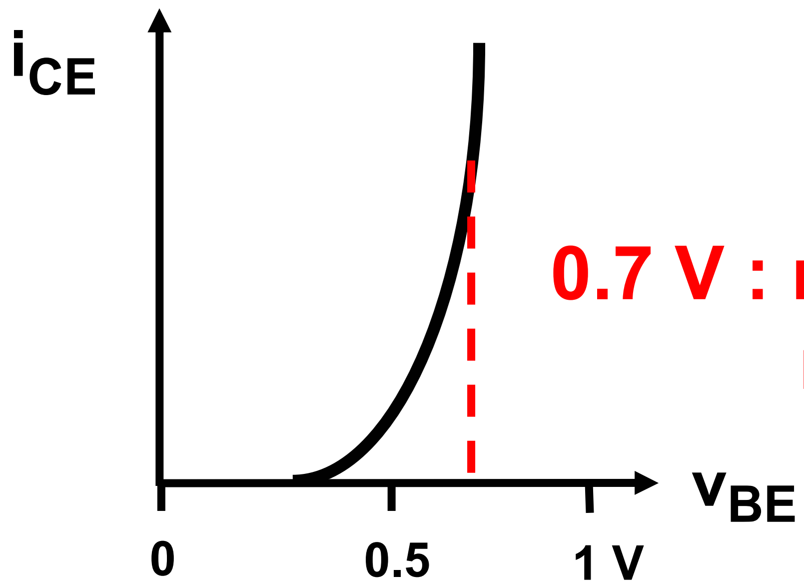
- Models of MOST transistors
- Models of Bipolar transistors
- Comparison of MOSTs & Bipolar transistors

□Bipolar transistors



Bipolar transistor I_{CE} versus V_{BE}





0.7 V : not more
not less

$$I_{CE} = I_S \exp \frac{V_{BE}}{kT/q}$$

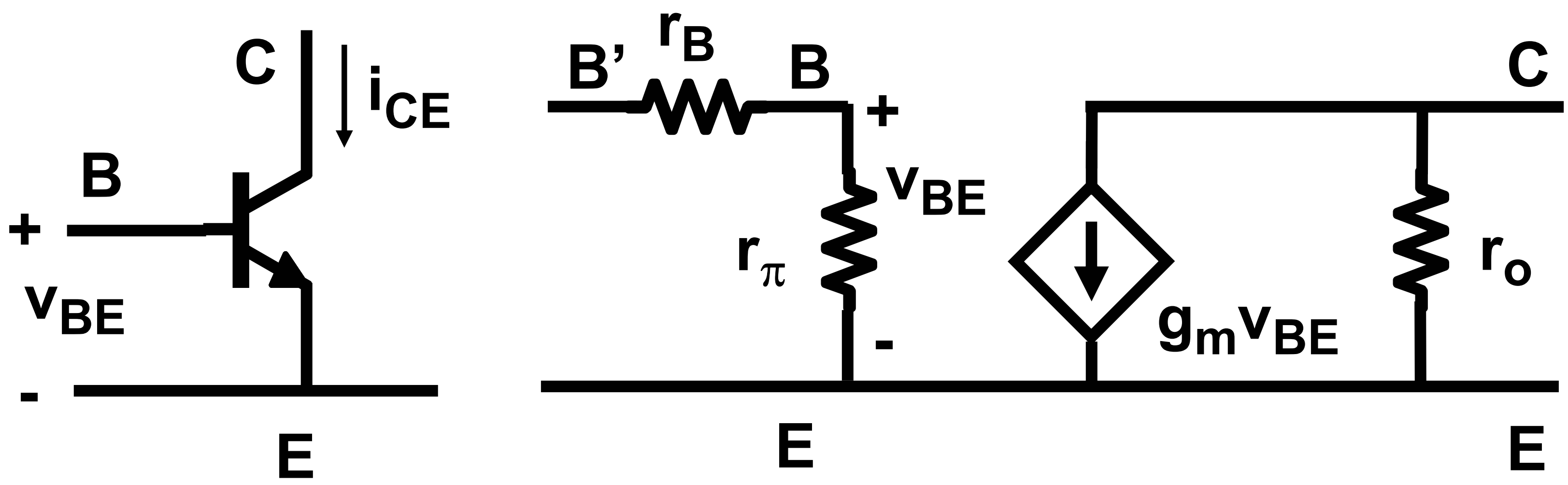
$I_S \approx 10^{-15}$ A kT/q = 26 mV at 300 K

$$I_{BE} = \frac{I_{CE}}{\beta}$$

is leakage current

$\beta \approx$ 10 … 1000

Bipolar transistor small-signal model : g_m & r_o



$$g_m = \frac{di_{CE}}{dv_{BE}} = \frac{I_{CE}}{kT/q}$$

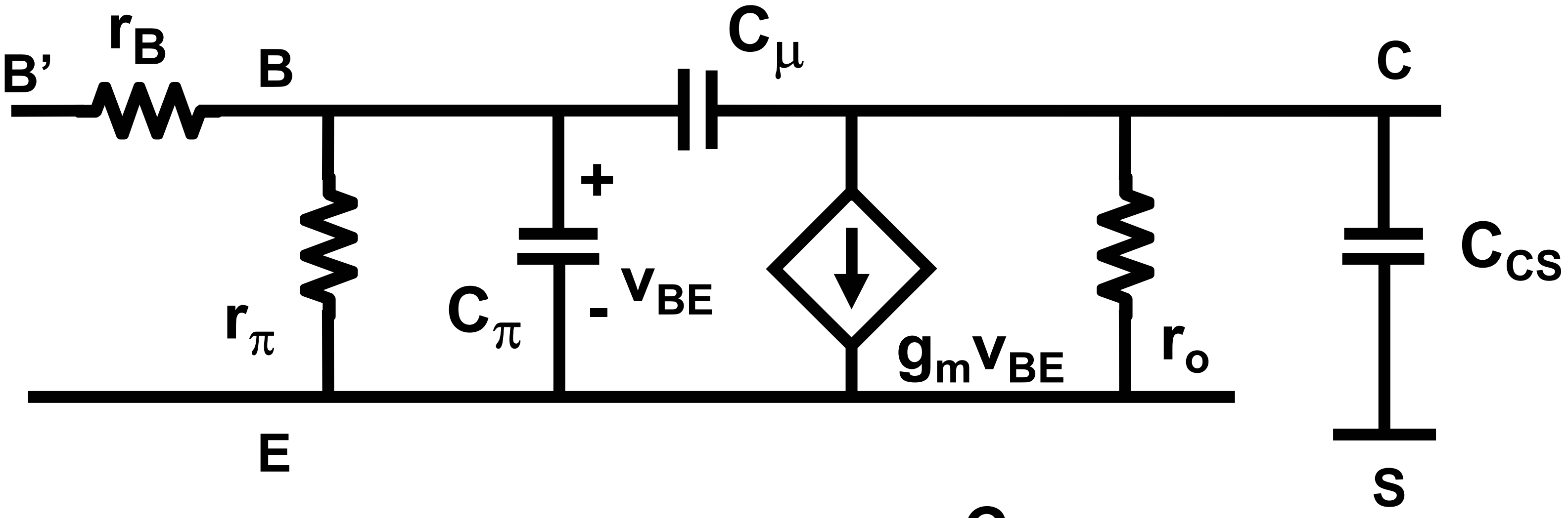
$$\frac{g_m}{I_{CE}} = \frac{1}{kT/q} \approx 40\ V^{-1}$$

$$r_\pi = \frac{dv_{BE}}{di_{BE}} = \beta\,\frac{dv_{BE}}{di_{CE}} = \frac{\beta}{g_m}$$

$$r_o = \frac{V_E}{I_{CE}}$$

$V_{En} \approx 20$ V
$V_{Ep} \approx 10$ V

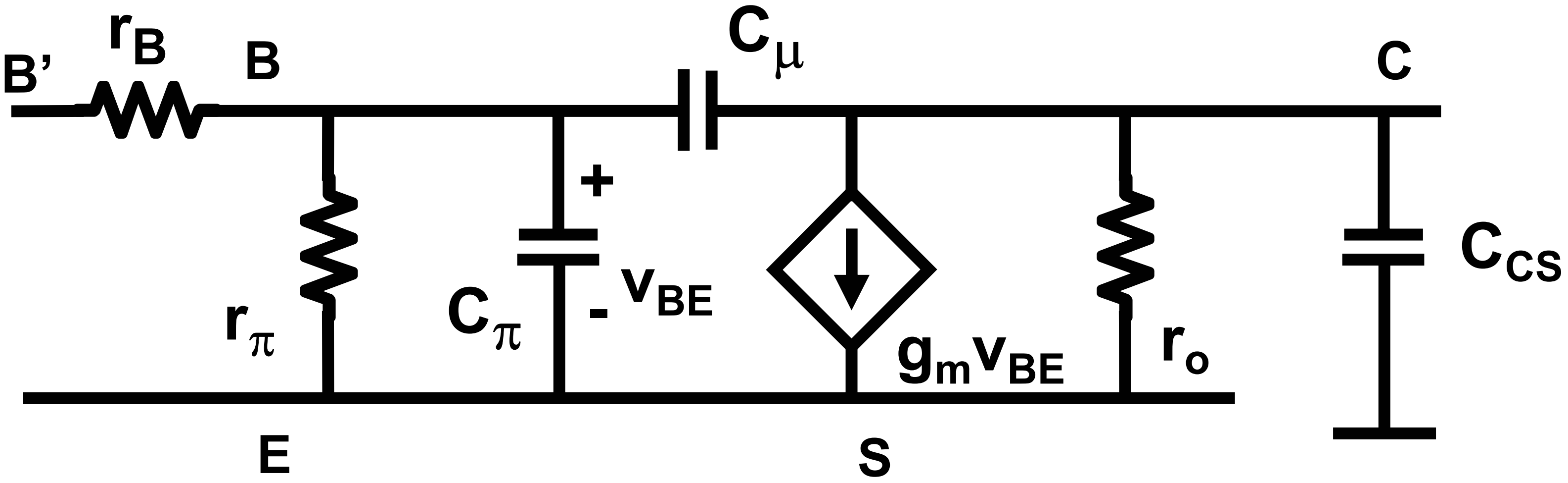
Bipolar transistor capacitance C_π



$$C_\pi = C_{jBE} + C_D \qquad C_{jBE} = \frac{C_{jBE0}}{\sqrt{1 + V_{BE}/\phi_{jE}}} \qquad \phi_{jE} \approx 0.7\ V$$

C_D is the diffusion capacitance

Diffusion capacitance C_D

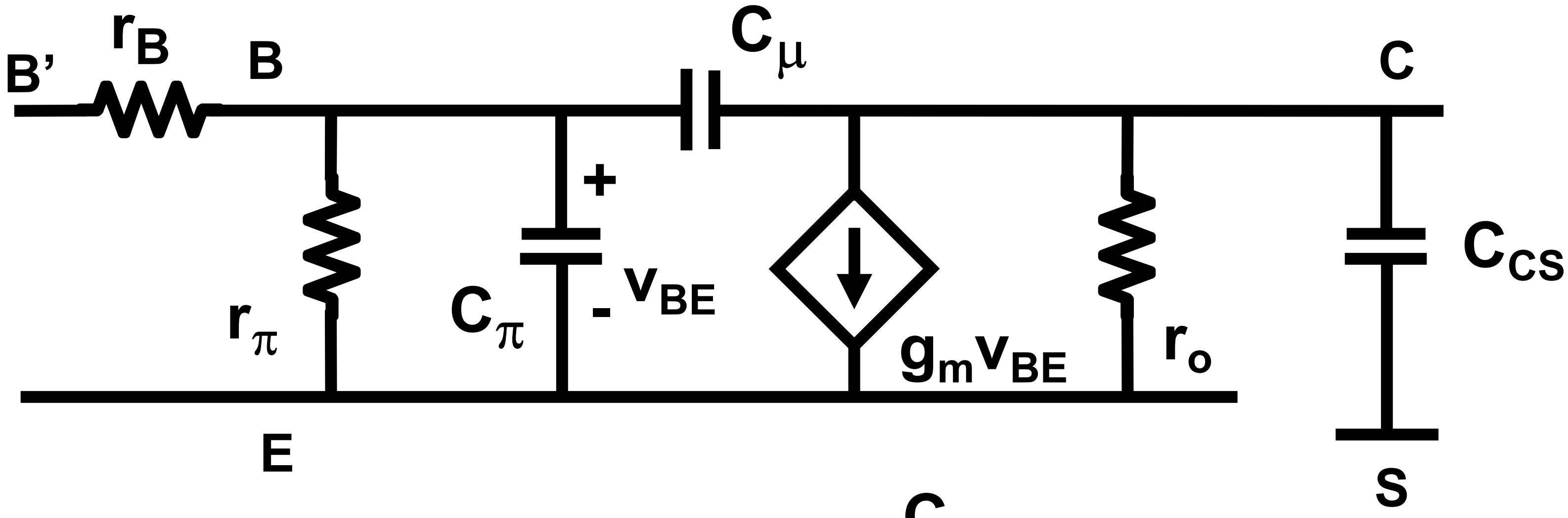


$$C_D = \frac{Q_B}{v_{BE}} = \tau_F \frac{di_{CE}}{dv_{BE}} = \tau_F g_m = \tau_F \frac{I_{CE}}{kT/q}$$

Base transit time $\tau_F = \dfrac{W_B^2}{2D_n}$ or now $\approx \dfrac{W_B}{v_{sat}}$

≈ 10 … 200 ps

Bipolar transistor capacitances C_μ & C_{CS}

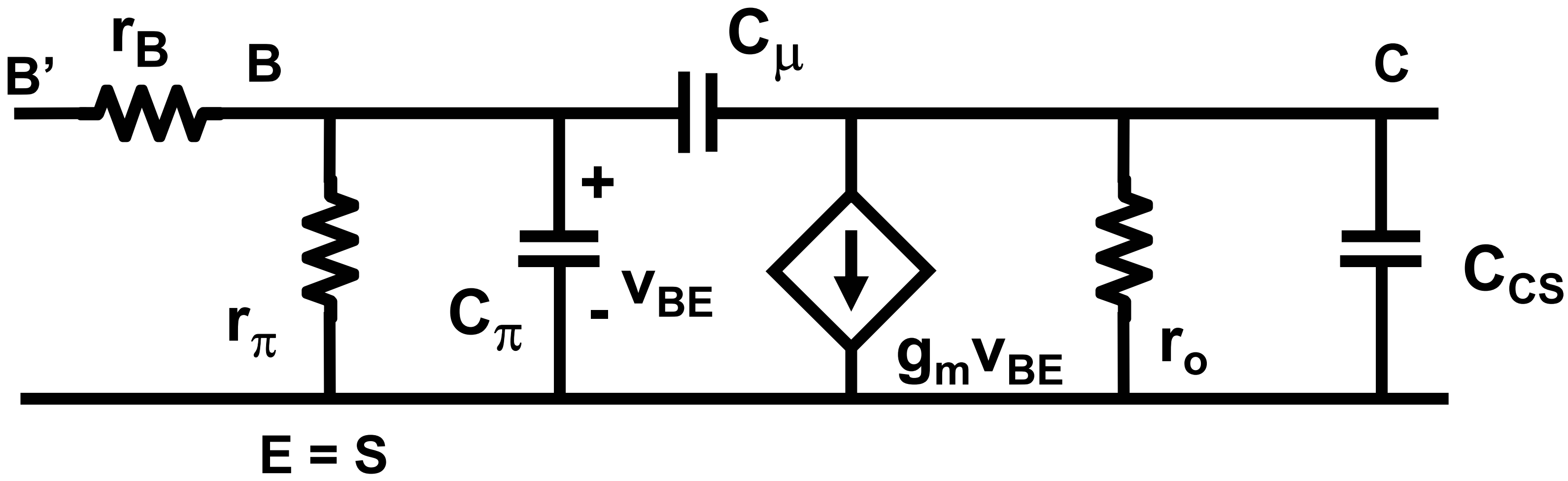


$$C_\mu = C_{jBC} \qquad C_{jBC} = \frac{C_{jBC0}}{\sqrt{1 + V_{BC}/\phi_{jC}}}$$

$$C_{CS} = C_{jCS} \qquad C_{jCS} = \frac{C_{jCS0}}{\sqrt{1 + V_{CS}/\phi_{jS}}}$$

$\phi_{jC} \approx \phi_{jS} \approx 0.5$ V

Bipolar transistor f_T



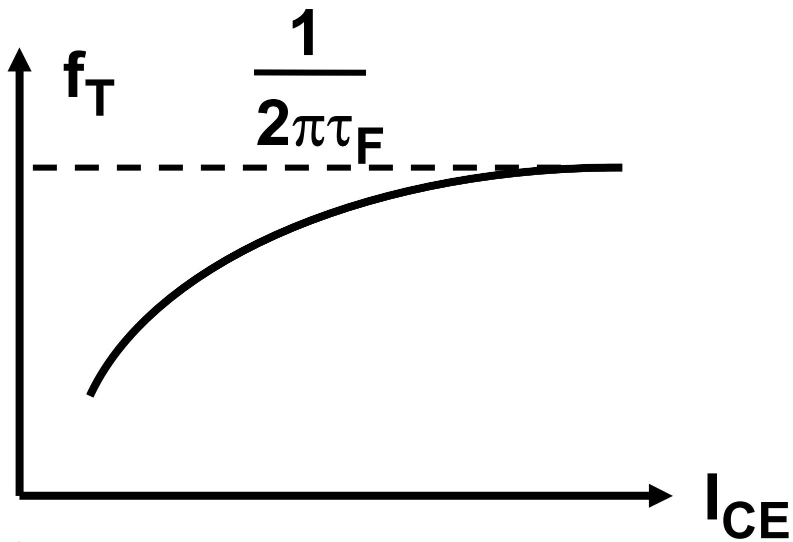
$$f_T = \frac{g_m}{2\pi\, C_\pi} = \frac{1}{2\pi} \frac{1}{\tau_F + \frac{C_{jBE}+C_\mu}{g_m}}$$

or $\approx \dfrac{v_{sat}}{2\pi\, W_B}$

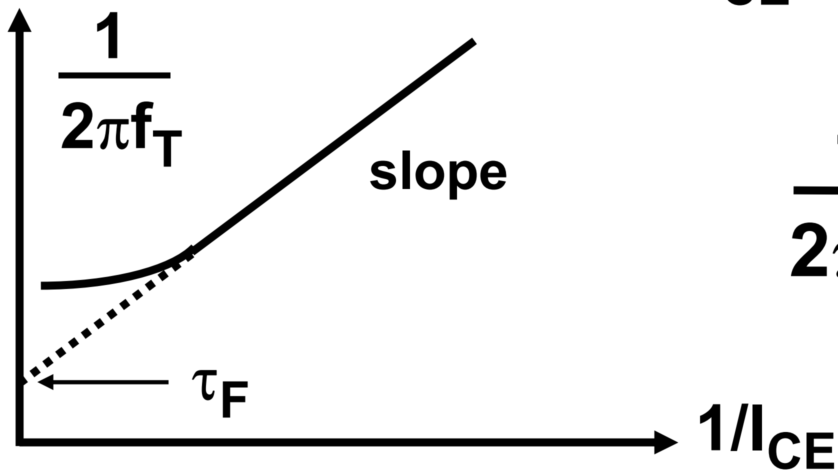
For a current drive !

$$f_{max} \approx \sqrt{f_T / 8\pi\, r_B C_\mu}$$

Bipolar transistor f_T versus I_{CE}



$$f_T = \frac{1}{2\pi}\,\frac{1}{\tau_F + \dfrac{C_{jBE}+C_\mu}{g_m}}$$



$$\frac{1}{2\pi f_T} = \tau_F + \underbrace{(C_{jBE} + C_\mu)\,\frac{kT}{q}}_{\text{slope}}\,\frac{1}{I_{CE}}$$

Single-page Bipolar transistor model

$$I_{CE} = I_S \exp \frac{V_{BE}}{kT/q}$$

$I_S \approx 10^{-15}$ A $kT/q = 26$ mV at 300 K

$$g_m = \frac{I_{CE}}{kT/q} \qquad r_o = \frac{V_E}{I_{CE}}$$

$V_{En} \approx 20$ V $V_{Ep} \approx 10$ V

$$f_T = \frac{1}{2\pi} \frac{1}{\tau_F + \frac{C_{je}+C_{jc}}{g_m}}$$

$$\text{or} \approx \frac{v_{sat}}{2\pi W_B}$$

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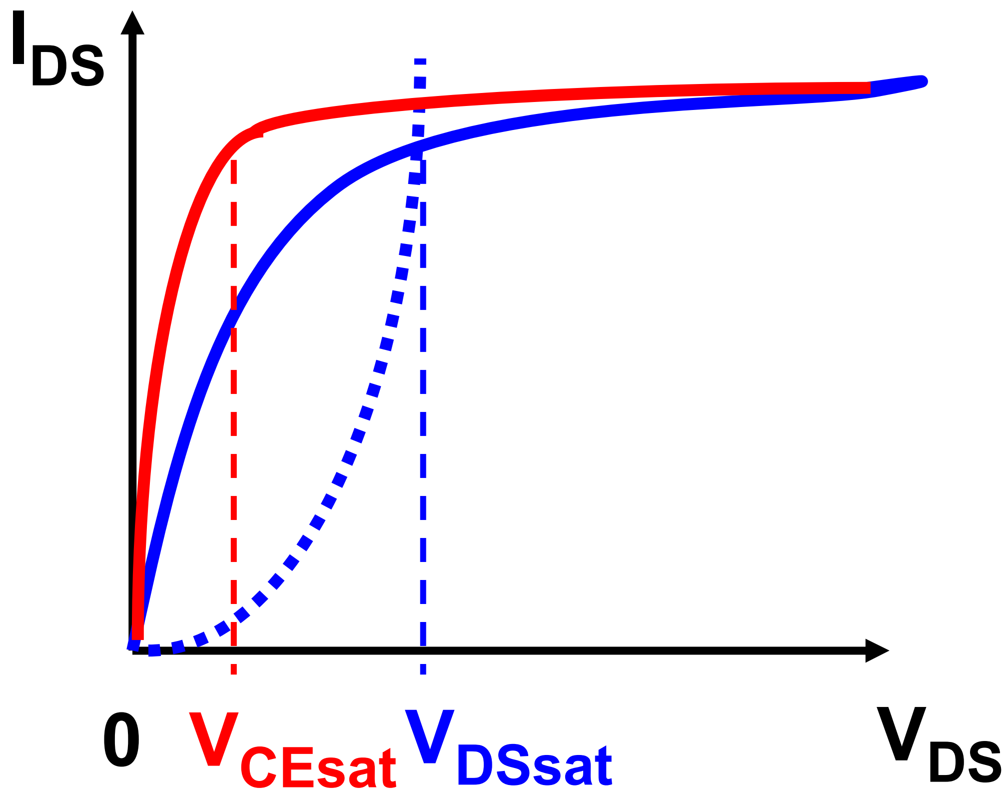
Comparison MOST - Bipolar

TABLE 2-8 COMPARISON OF MOSTS AND BIPOLAR TRANSISTORS

	Specification		MOST	Bipolar transistor	
1.	I_{IN}		0	I_C/β	β ?
	R_{IN}		∞	$r_\pi + r_B$	
2.	$V_{DS\,sat}$		$V_{GS} - V_T = \sqrt{\dfrac{I_{DS}}{K'W/L}}$	few kT/q	
3.	$\dfrac{g_m}{I}$	wi	$\dfrac{1}{nkT/q}$	$\dfrac{1}{kT/q}$	$n = 1 + \dfrac{C_D}{C_{ox}}$
		si	$\dfrac{2}{V_{GS} - V_T}$	$\dfrac{1}{kT/q}$	4… 6 x
		vs	$\dfrac{1}{V_{GS} - V_T}$	$\dfrac{1}{kT/q}$	

Ref. Laker Sansen Table 2-8

Comparison MOST - Bipolar : minimum V_{DS}



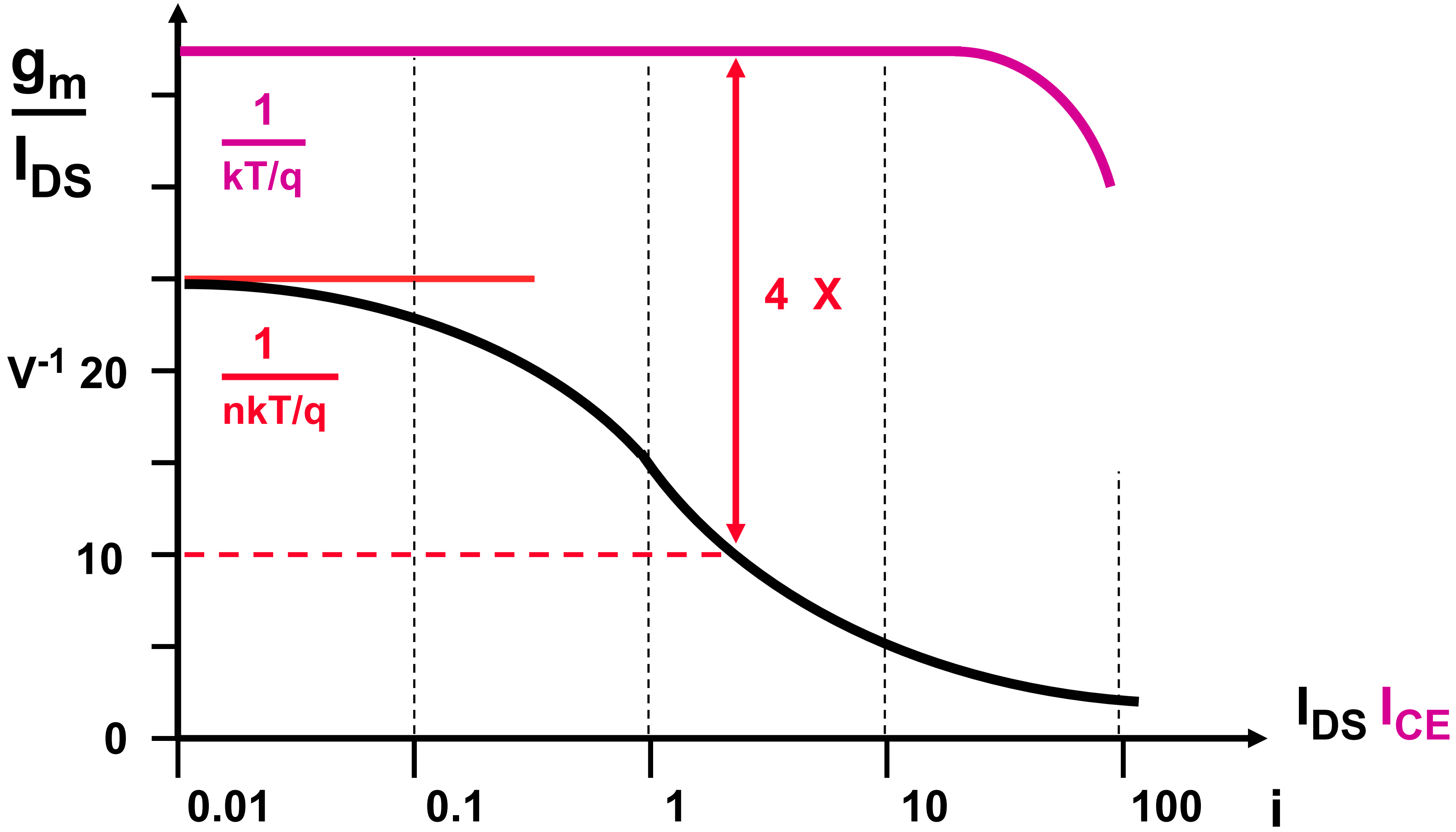
$$V_{DSsat} \approx V_{GS} - V_T$$

$$V_{GS} - V_T \approx \sqrt{\frac{I_{DS}}{K'_n \frac{W}{L}}}$$

$$V_{CEsat} \approx kT/q\text{'s}$$

Ref. Laker - Sansen Table 2-8

Comparison MOST - Bipolar : g_m/I_{DS} ratio



Design plan for g_m :

$$I_{DS} = K'_n \frac{W}{L} (V_{GS}-V_T)^2$$

$$g_m = 2K'_n \frac{W}{L} (V_{GS}-V_T) = 2 \sqrt{K'_n \frac{W}{L} I_{DS}} = \frac{2 I_{DS}}{V_{GS}-V_T}$$

4 variables with 2 equations >> 2 free variables

Choose $V_{GS}-V_T$ and L !

Comparison MOST - Bipolar

4.	Design planning		$L, V_{GS} - V_T$	kT/q
5.	I-range		1 decade	7 decades
6.	Max f_T	low I	C_{GS}, C_{GD}	C_{jEt}, C_{μ}
		high I	v_{sat}/L_{eff}	v_{sat}/W_B
7.	Noise $\overline{dv_i^2}$	Therm.	$4kT\left(\frac{2/3}{g_m} + R_G\right)$	$4kT\left(\frac{1/2}{g_m} + R_B\right)$
		$1/f$	$10\times$	
	Offset		$10\times$	

$v_{sat} \approx 10^7$ cm/s

Ref. Laker Sansen Table 2-8

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- **Models of MOST transistors**
- **Models of Bipolar transistors**
- **Comparison of MOSTs and Bipolars**

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

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