Efficient On-Line Schedulability Tests and Configuration Selection

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Contents

- Motivation
- Configuration Selection
- Schedulability Test for the Liu&Layland Model
- Schedulability Test for the Multiframe Model
- Performance Evaluation
- Conclusion

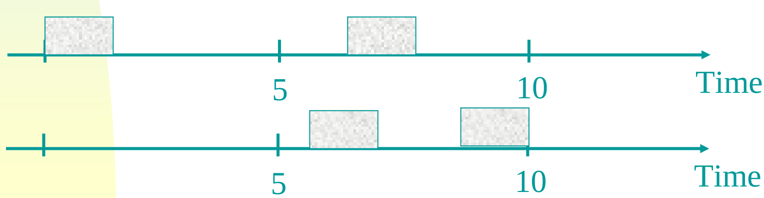
Motivation

- In there a systematic way in selecting a better configuration for processes?
- If overload is detected, what should we do?
- How to schedule processes whose timing constraints change in reaction to the environment?

What to do when overload is detected?

Graceful Degradation! But how?

- Load Shedding – kill less important processes!
- Relax timing constraint briefly!
  - E.g., instead of processing a sporadic interrupt within 5 seconds, promise to process 2 interrupts within 10 seconds!



Another thought in relaxing timing constraints!

- Load Scaling
  - Unschedulable configurations

    $\{(\tau_A, 1.5, 3), (\tau_B, 2, 4)\}$

    $\{(\tau_A, 1.5, 3), (\tau_B, 2.5, 5)\}$
  - Schedulable configurations

    $\{(\tau_A, 1.5, 3), (\tau_B, 3, 6)\}$

    Furthermore,

    $\{(\tau_A, 1.5, 3), (\tau_B, 1.5, 3)\}$
  - τ_A and τ_B become schedulable when their periods are harmonically related!!

Questions: How to choose periods?

- Configuration Selection Problem:
  - Given a set of configurations, choose a schedulable configuration!
  - Period Assignment Problem:
    - Given a set of adaptive processes, choose a schedulable configuration!
      - An adaptive process may change its timing constraints!

Introduction

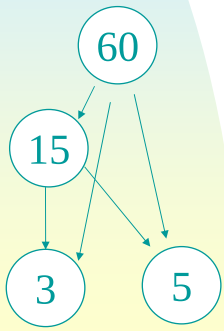
- Needs of Schedulability Tests
  - Performance Guarantee
  - Resource Reservation
  - Open System Architecture
  - etc
- Approaches:
  - Achievable Utilization Factor
  - Rate Monotonic Analysis (RMA)

Introduction

- A Sufficient Schedulability Condition
  - Liu & Layland $n(2^{1/n} - 1)$
    (n = process #)
  - Kuo & Mok $k(2^{1/k} - 1)$
    (k = fundamental frequency #)
  - Mok & Chen $r * n((\frac{r+1}{r})^{1/n} - 1)$
    (r = min (c_i^0/c_i^1))
  - *and various tests by Han, et al.*
- Sufficient and Necessary Schedulability Conditions
  - Rate Monotonic Analysis (RMA)

Introduction

- Motivation
  - On-line schedulability tests
  - Better precision



	τ_1	τ_2	τ_3	τ_4
(c_i, p_i)	(1,3) U1=0.333	(1,5) U2=0.533	(2,15) U3=0.666	(8,60) U4=0.8
Op 1 (r=1.875)	p1' = 1.875 U1=0.533	p2' = 3.75 U2=0.8	p3' = 15 U3=0.933	p4' = 60 U4=1.066
Op 2 (r=2.5)	p1' = 2.5 U1=0.4	p2' = 5 U2=0.6	p3' =10 U3=0.8	p4' = 40 U4=1.0
Op 3 (r=3)	p1' = 3 U1=0.333	p2' = 3 U2=0.666	p3' = 12 U3=0.833	p4' = 48 U4=1.0

Kuo-Mok $k(2^{1/k}-1)$

Han-Tyan $p_i' = r \cdot 2^{\lceil \log_2(p_i/p_1) \rceil}$

Our Approach

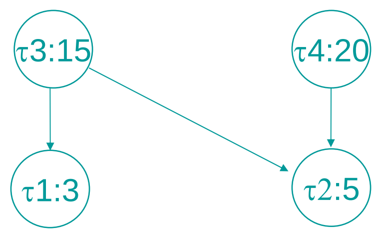
- Exploit the harmonic relationship of task periods to improve existing schedulability tests:
  - Liu & Layland Process Model
  - Multiframe Process Model
- Efficient for on-line implementation.
- Effective for heavy CPU utilizations!

Definitions

- Critical Instant
- Critical Interval
- Utilization Factor

$$U = \sum_{i=1}^{n} \frac{c_i}{p_i}$$

- Division Graph
- Root



Definitions

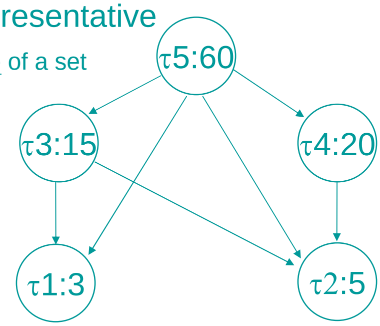
- Offspring Set

  e.g., {τ1, τ2, τ4} and {τ1, τ2, τ3, τ4, τ5} are offspring sets of τ5.

- Reduced Set and RS-Representative

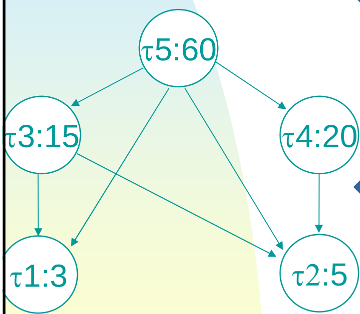
  A process τ is a RS-representative of a set {τ1, τ2, τ3} if the period of τ is 15, and the utilization factor is equal to the sum of the utilization factors of τ1, τ2, and τ3 .

  {τ1, τ2, τ3} is a reduced set of τ .



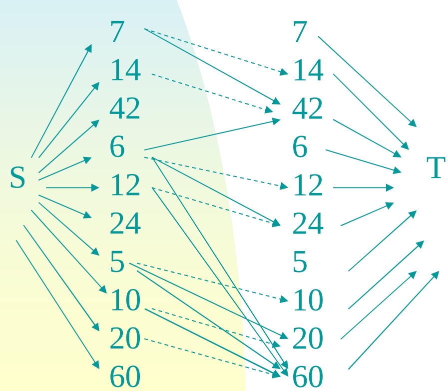
Schedulability Tests for the Liu&Layland Model

- Computing the smallest harmonic base
  - Definition: Division Graph
    - An irreflexive, asymmetric, transitive, acyclic directed graph to represent the divisibility relation among a set of real numbers
  - Theorem 9 [KM:97] Given a set of n processes, there exists a corresponding division graph G. If K is the minimum number that G can be decomposed into vertex-disjoint linear paths, then its least upper bound of utilization factor is $K(2^{1/K}-1)$.



Schedulability Tests for the Liu&Layland Model

An $O(N^{5/2})$ Algorithm



- Definition: Minimum Linear Covering [KM:97] – Given an acyclic directed graph G, the problem is to find the smallest integer K such that the vertices of G are partitioned into K vertex-disjoint linear paths. K is the minimum linear covering number of G.

Schedulability Tests for the Liu&Layland Model

More Precise Schedulability Tests?

Schedulability Tests for the Liu&Layland Model

- Theorem 1 [Lehoczky, Sha, Ding 89]

Process τ_i in a set of periodic processes scheduled by a fixed-priority-driven preemptive scheduling algorithm will always meets its deadline for all processes phases if and only if there exists a pair *(k,m) in R_i* such that

$$\sum_{j<i} (c_j \left\lceil \frac{mp_k}{p_j} \right\rceil) + c_i \leq mp_k$$

where

$$R_i = \{(k,m) \mid 1 \leq k \leq i, m = 1,2,\Lambda , \left\lfloor \frac{p_i}{p_k} \right\rfloor \}$$

Schedulability Tests for the Liu&Layland Model

- Lemma 1

Suppose that T_{i-1} is schedulable. Let τ_j be any process in T_i, TO_j a subset of an offspring set of τ_j in T_i which includes τ_j, and τ the RS-representative of TO_j. Let process τ' be the process with the largest period in $(T_i - TO_j \cup \{\tau\})$, where the period and computation requirements of τ' are p_i and c, respectively. If there exists a pair $(k,m) \in R'$ such that

$$\sum_{\tau_x \in (T_i - TO_j \cup \{\tau\} - \{\tau'\})} (c_x \left\lceil \frac{mp_k}{p_x} \right\rceil) + c \le mp_k$$

where $R' = \{(k,m) \mid \tau_k \in (T_i - TO_j \cup \{\tau\}), m = 1,2,\ldots, \left\lfloor \frac{p_i}{p_k} \right\rfloor \}$

then T_i (including τ_j) is schedulable.

Schedulability Tests for the Liu&Layland Model

- Theorem 2

Suppose that T_{i-1} is schedulable, and ST_i is a non-empty subset of T_i. For each process τ_j in ST_i, let TO_j be a subset of an offspring set of τ_j in T_i such that $TO_j \cap ST_i = \{\tau_j\}$, and $TO_j \cap TO_k = \{\}$ for any two distinct processes τ_j and τ_k in ST_i. For each process τ_j in ST_i, τ'_j is the RS-representative of TO_j. Let process τ' be the process with the largest period in

$$T_i' = (T_i - \bigcup_{j \in ST_i} TO_j \cup \{\tau'_j \mid \tau'_j \text{ is the RS-representative of } TO_j, \text{for every } \tau_j \in ST_i\})$$

where the period and the computation requirements of τ' are p_i and c, respectively. If there exists a pair $(k,m) \in R'$ such that

$$\sum_{\tau_x \in (T_i' - \{\tau'\})} (c_x \left\lceil \frac{mp_k}{p_x} \right\rceil) + c \le mp_k$$

where $R' = \{(k,m) \mid \tau_k \in T_i', \mathrm{m} = 1, 2, \ldots, \left\lfloor \frac{\mathrm{p_i}}{\mathrm{p_k}} \right\rfloor \}$ then T_i is schedulable.

Schedulability Tests for the Liu&Layland Model

- Theorem 3 [Liu&Layland 73]

  A set of *n* periodic processes is schedulable if the total utilization factor of the process set is no larger than

  $$n(2^{1/n} - 1)$$

- Theorem 4

  Suppose that T_{i-1} is schedulable. Let *k* be the number of roots in T_i. If the total utilization factor of T_i is no larger than

  $$k(2^{1/k} - 1)$$

  then T_i is schedulable.

A Schedulability Test Algorithm

- Step 1: i = 1;
- Step 2: If there are K roots in T_i

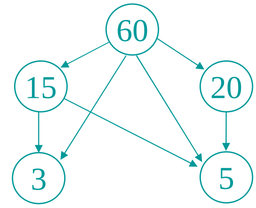
  and $U_i \leq K(2^{1/K}-1)$,

  Then τ_i is schedulable

  Else the schedulability of τ_i is not guaranteed; Exit;
- Step 3: i = i + 1;
- Step 4: Goto Step 1 unless i > n;

Complexity: $O(n^2)$

Schedulability Tests for the Liu&Layland Model



Example : $F(n) = n(2^{1/n} - 1)$

F(1) = 1

F(2) = 0.8284

F(3) = 0.7798

F(4) = 0.7568

F(5) = 0.7435

τ_i	1	2	3	4	5
p_i	3	5	15	20	60
c_i	1	1	2	3	8
U_i	0.3333	0.5333	0.6666	0.8166	0.95

- Theorem 5 [Kuo&Mok 91]

  A set of periodic processes with k fundamental frequencies is schedulable if the total utilization factor of the process set is no larger than

$$k(2^{1/k} - 1)$$

Extension: Multiframe Model

- Why the Multiframe model?
  - Modeling of processes with varying timing constraints!
  - Modeling of processes with skipping of process executions in consecutive periods.
- Goal:
  - Extend the idea of reduced-set-based schedulability tests to the multiframe model to have a more precise test!

Schedulability Tests for the Multiframe Model

- Definition [Mok and Chen 96]
  - Multiframe Process

    A multiframe real-time process τ_i is a tuple (Γ_i, p_i), where Γ_i is an array of N_i execution times $(c^0_i, c^1_i, \ldots, c^{Ni-1}_i)$ for some $N_i \geq 1$, and p_i is the period of τ_i.
  - Remark :

    Let c^0_i be the maximum in an array of execution times $(c^0_i, c^1_i, \ldots, c^{Ni-1}_i)$. c^0_i is called the peak execution time of τ_i .
  - AM Multiframe Process

    An array $(c^0_i, c^1_i, \ldots, c^{Ni-1}_i)$ is said to be AM (Accumulative Monotonic) if

$$\sum_{k=0}^{j} C_i^{k \bmod N_i} \geq \sum_{k=x}^{x+j} C_i^{k \bmod N_i}, 1 \leq x \leq (N_i - 1), 1 \leq j \leq (N_i - 1)$$

A multiframe process $\tau_i = \{\Gamma_i = (c^0_i, c^1_i, \ldots, c^{Ni-1}_i), p_i\}$ is said to be AM if its array of execution times is AM .

Schedulability Tests for the Multiframe Model

- Critical Instance of AM Multiframe Process [Mok&Chen 96]

The critical instance of an AM multiframe process is the beginning of the period when its peak execution time is requested simultaneously with the peak execution times of all higher priority processes.

Schedulability Tests for the Multiframe Model

- Reduced Set and RS-Representative of AM Multiframe Process

Suppose that $\tau = \{\Gamma = (c^0, c^1, \ldots, c^{N-1}), p\}$ is a multiframe periodic process with period p and an array Γ of computation requirements.

A set of *n* AM multiframe periodic processes $\{\tau_1, \tau_2, \ldots, \tau_n\}$, where process τ_i has period p_i and an array $\Gamma_i = (c^0_i, c^1_i, \ldots, c^{Ni-1}_i)$ of N_i computation requirements, is a reduced set of τ if

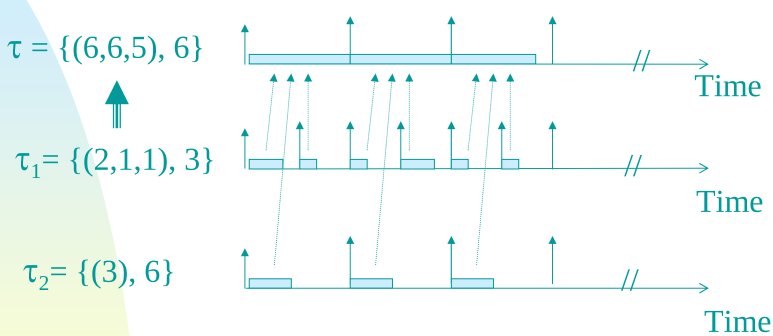
$$N = LCM(N_i)$$

$$p_i \mid p, p = \max_{i=1}^{n} p_i$$

$$C^j = \sum_{i=1}^{n} \left(\sum_{k=0}^{(p/p_i)-1} C_i^{(j(p/p_i)+k) \bmod N_i}\right)$$

τ is called the RS-representative of the reduced set.

Schedulability Tests for the Multiframe Model



- Claim 1: The RS-representative of a set of AM multiframe periodic processes is an AM multiframe periodic process.

Schedulability Tests for the Multiframe Model

- Theorem 6 [Mok&Chen 96]

  For the preemptive fixed priority scheduling policy, an AM multiframe process is schedulable if it is schedulable at its critical instance

- Theorem 7

  Process τ_i in a set of AM multiframe periodic processes scheduled by a fixed priority-driven preemptive scheduling algorithm will always meet its deadline for all process phases iff there exists a pair $(k,m) \in R_i$ such that

$$\sum_{j<i} \left(\sum_{x=0}^{\left\lceil \frac{mp_k}{p_j} \right\rceil - 1} C_j^{x \bmod N_j} \right) + C_i^0 \le mp_k$$

where $R_i = \{(k,m) \mid 1 \le k \le i, m = 1,2,\Lambda, \left\lfloor \frac{p_i}{p_k} \right\rfloor\}$

Schedulability Tests for the Multiframe Model

- Lemma 2

  Suppose that the AM multiframe process set T_{i-1} is schedulable. Let be τ_j any multiframe process in T_i, TO_j any subset of an offspring set of τ_j, and $\tau_j = \{(c^0, c^1, \ldots, c^{N-1}), p\}$ the RS-representative of TO_j. Let process τ' be the process with the largest period in $(T_i - TO_j \cup \{\tau\})$, where the period and the array computation requirements of τ' are p_i and $\Gamma' = (c'^0, c'^1, \ldots, c'^{N'-1})$, respectively. If there exists a pair $(k, m) \in R'$ such that

$$\sum_{\tau_x \in (T_i - TO_j \cup \{\tau\} - \{\tau'\})} \left(\sum_{y=0}^{\left\lceil \frac{mp_k}{p_x} \right\rceil - 1} C_x^{y \bmod N_x} \right) + C'^0 \le mp_k$$

where $R' = \{(k,m) \mid \tau_k \in (T_i - TO_j \cup \{\tau\}), m = 1,2,\Lambda, \left\lfloor \frac{p_i}{p_k} \right\rfloor\}$, then AM multiframe process set T_i is schedulable.

Schedulability Tests for the Multiframe Model

- Theorem 8

  Suppose that the AM multiframe process set T_{i-1} is schedulable, and ST_i is a non-empty subset of the AM multiframe process set T_i. For each process $\tau_j \in T_i$, let TO_j be a subset of an offspring set of τ_j in T_i such that $TO_j \cap ST_i = \{\tau_j\}$, and $TO_j \cap TO_k = \{\}$ for any two distinct processes τ_j and τ_k in ST_i. For each process $\tau_j \in ST_i$, τ'_j is the RS-representative of TO_j. Let process τ' be the process with the largest period in

$$T_i' = (T_i - \bigcup_{j \in ST_i} TO_j \cup \{\tau'_j \mid \tau'_j \text{ is the RS-representative of } TO_j, \text{for every } \tau_j \in ST_i\})$$

  where the period and the array of computation requirements of τ' are p_i and $\Gamma' = (c'^0, c'^1, \ldots, c'^{N'-1})$, respectively. If there exists a pair *(k, m)* $\in R'$ such that

$$\sum_{\tau_x \in (T_i' - \{\tau'\})} \Big(\sum_{y=0}^{\lceil \frac{mp_k}{p_x} \rceil - 1} C_x^{y \bmod N_x} \Big) + C'^0 \le mp_k$$

  where $R' = \{(k,m) \mid \tau_k \in T_i', \mathrm{m} = 1, 2, \ldots, \lfloor \frac{\mathrm{p_i}}{\mathrm{p_k}} \rfloor\}$ then T_i is schedulable.

Schedulability Tests for the Multiframe Model

- Definition: Peak Utilization Factor

  The peak utilization factor U_m of an AM multiframe process set $T = \{\tau_1, \ldots, \tau_n\}$ is equal to $\sum_{i=1}^{n} c_i^0 / p_i$

  where c^0_i and p_i are the peak execution time and period of τ_i, respectively.

- Theorem 9 [Mok&Chen96]

  Let $r = \min_{i=1}^{n} (C_i^0 / C_i^1)$.

  For process sets of size n, the bound on peak utilization factor is given by $r \cdot n \cdot \left(\frac{r+1}{r}^{1/n} - 1 \right)$

Schedulability Tests for the Multiframe Model

- Theorem 10

  Suppose that T_{i-1} is schedulable. Let k be the number of roots in T_i. If the total peak utilization factor of T_i is no larger than

  $$r \cdot k \cdot \left(\left(\frac{r+1}{r}\right)^{1/k} - 1 \right)$$

  then T_i is schedulable, where r is calculated based on the RS-representative set of T_i.

- Remark :

  There is no way to directly compare the bounds on the peak utilization factors derived by Theorems 9 and 10 because r might be different in the original and transformed process sets.

Performance Evaluation

- Polynomial-Time Schedulability Tests under Comparison:
  - Liu&Layland Model
    - Liu & Layland 73

      (n = process #)
    - Kuo & Mok 91

      (k = fundamental frequency #)
    - Han & Tyan 97 (DCT+Sr)
    - Root/Reduced-Set Method
  - Multiframe Model
    - Mok & Chen 96

      (r = min (c_i^0/c_i^1))
    - Han & Tyan 98 (DCT+Sr)
    - Root/Reduced-Set Method

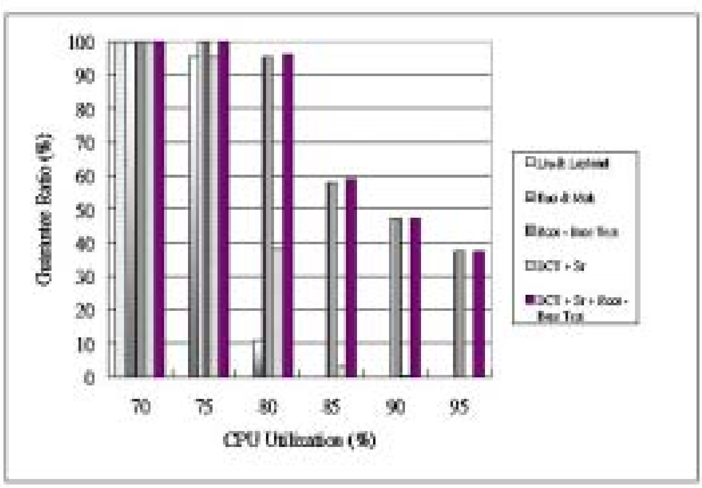
Performance Evaluation

- Performance Metrics
  - Guarantee Ratio:

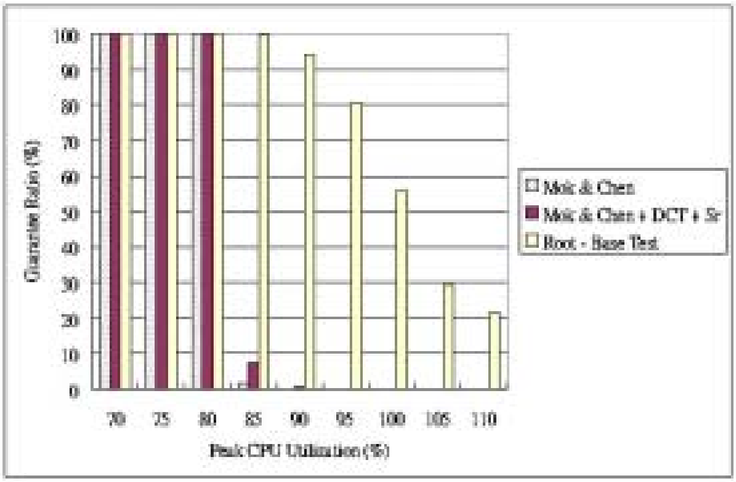
$$\frac{\text{(number of guarantee schedulable process sets)}}{\text{(number of process sets)}}$$

- Data Sets
  - Number of processes per process set: randomly chosen between 10 and 30.
  - Fundamental frequencies:
    - 1/4 ~ 1/10 of the number of processes.
    - Probability of assigning i fundamental frequencies to a process is $(1/2)^{i-1}$. Random assignment!
  - Utilization factor ranges from 70% ~ 95%.
    - 70% ~ 110% for multiframe process sets.
    - $C^j_i = C^0_i / r^j_i$ for ri in (2, 5), for each multiframe process τ_i.
  - 400 process sets per utilization factor were tested

Liu&Layland Model

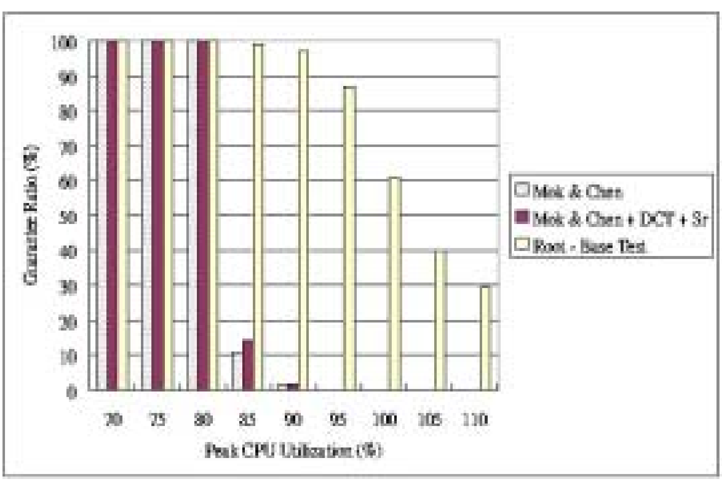


Multiframe Model



When $r \in (2, 5)$!

Multiframe Model



When $r \in (2, 10)$!

Conclusion

- **Summary**
  - Provides efficient on-line schedulability tests which consider harmonic relationship of process periods and the variance of computation times in different periods.
  - Provide better precision in identifying schedulable process sets, even under heavy CPU utilization.

- **Future research**
  - Extend the reduced-set methodology to analyze the schedulability of soft and firm real-time process sets
  - A process set mixed with hard, soft, and firm real-time processes.
  - Generalize the results to RMA-based schedulability tests to speed up their performance.