A Generalized Approach for the Acceleration and Deceleration of Industrial Robots and CNC Machine Tools

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***Abstract*—Many techniques for the acceleration and deceleration of industrial robots and computer numerical control (CNC) machine tools have been proposed in order to make industrial robots and CNC machine tools perform given tasks efficiently. Although the techniques selecting polynomial functions can generate various acceleration and deceleration characteristics, the major problem is the computational load. The digital convolution techniques are more efficient than the techniques selecting polynomial functions. However, neither velocity profiles of which the deceleration characteristics is independent from the acceleration characteristics nor those of which the acceleration interval is different from the deceleration interval can be generated by the digital convolution techniques. This paper proposes a generalized approach for generating velocity profiles that cannot be generated by the digital convolution techniques. According to the desired characteristics of acceleration and deceleration, each set of coefficients is calculated and is stored. Given a moving distance, and acceleration and deceleration intervals, a velocity profile having the desired characteristics of acceleration and deceleration can be efficiently generated by using these coefficients. Several velocity profiles generated by the proposed technique will be applied to one single-axis control system.**

***Index Terms*—Acceleration and deceleration, computer numerical control machine tool, industrial robot, velocity profile.**

I. Introduction

THE demand for better accuracy in the manufacturing of complicated parts and the desire to increase productivity have developed industrial robot and computer numerical control (CNC) systems so that industrial robots and CNC machine tools move more accurately and more quickly. Since the combined characteristics of the control and the machine tool determine the final accuracy and productivity of industrial robot and CNC systems, there are many factors to consider for improving these quantities. One of the important factors is efficiently generating motion profiles that have the desired acceleration and deceleration characteristics that in turn are determined according to given machining tasks. Many researchers have proposed techniques for generating motion profiles of industrial robots and CNC machine tools. One of them is generating position trajectories by the selection of polynomial functions [1]. This technique can generate so many kinds of acceleration and deceleration characteristics and, furthermore, can make the characteristics of deceleration be independent from that of acceleration. In order to achieve high-performance motion control, the motion profiles must be matched to the system limits such as the maximum acceleration and the maximum velocity. If the motion profiles matched to the system limits are generated by the selection of polynomial functions, the amount of computation becomes larger as the order of the polynomial becomes higher. That is, the computational load is increased almost exponentially with the order of polynomial. Thus, the position trajectories by the selection of polynomial functions that are matched to the system limits are confined to some fixed patterns of which the velocity profiles are typically trapezoidal. However, since trapezoidal velocity profiles have large jerk, smooth velocity profiles that are matched to the system limits and have small jerk have been preferred for industrial robots and CNC machine tools [2]–[5]. If position trajectories of which the velocity profiles are smooth are generated by the selection of polynomial functions, it requires a lot of computations. Due to time constraints, it is very difficult to apply the technique selecting polynomial functions to controlling industrial robot and CNC machine tools. Other previous techniques for generating velocity profiles are based on a digital convolution [5]–[9]. These techniques are much more efficient than the techniques selecting polynomial functions and are easily implemented by hardware. But, in the velocity profiles generated by these techniques, the acceleration interval is always the same as the deceleration interval and the characteristics of deceleration are dependent on that of the acceleration. Thus, some velocity profiles that are useful for industrial robots and CNC machine tools cannot be generated by these techniques [4].

In this paper, a generalized approach for the acceleration and deceleration of industrial robots and CNC machine tools is proposed. The proposed technique is as simple and efficient as the techniques based on a digital convolution and can generate velocity profiles which have more various characteristics of acceleration and deceleration than the techniques based on a digital convolution can. That is, the proposed technique can generate velocity profiles of which the deceleration characteristics are independent from the acceleration characteristics. First, some coefficients are calculated and are stored according to the acceleration characteristic, the deceleration characteristic, and the acceleration interval, and the deceleration interval. Then, given

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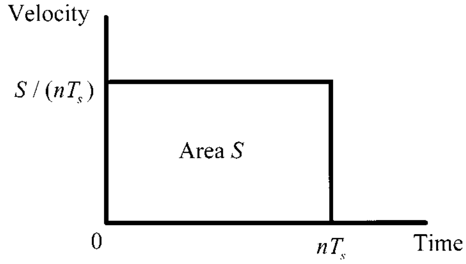

Fig. 1. A rectangular velocity profile for moving a distance S.

a desired moving distance, a velocity profile that has the desired characteristics is generated by calculating the position increment during each sampling time. The position increment during each sampling time is calculated by multiplying the stored coefficients with the product of one sampling time and the velocity after acceleration.

In Section II, existing techniques for generating velocity profiles of industrial robots and CNC machine tools will be explained. In Section III, it will be explained how to generate a desired velocity profile by the proposed technique. The proposed technique and other existing techniques will be compared. In Section IV, experiments will be performed.

II. Previous Techniques

For the illustration of the previous techniques, let us consider one single-axis control system of which the maximum velocity, the maximum acceleration, and the sampling time are $V_{\max}$, $A_{\max}$, and T_s respectively. If this system moves the given distance S at the maximum velocity $V_{\max}$, then the movement time T_1 in the rectangular velocity profile will be

$$T_1 = \frac{S}{V_{\max}} = pT_s. \tag{1}$$

Selecting an integer n which is the smallest integer among integers which are equal to or greater than p, the resulting rectangular velocity profile is constructed as in Fig. 1. The velocity and position equations for this profile are

$$V_0(t) = \frac{S}{nT_s}, \qquad 0 \le t \le nT_s \tag{2}$$

$$P_0(t) = \frac{S}{nT_s}t, \qquad 0 \le t \le nT_s. \tag{3}$$

The position increment during each sampling time is given by

$$\delta P_0(kT_s) = P_0(kT_s) - P_0((k-1)T_s) = \frac{S}{n} \quad 1 \le k \le n \tag{4}$$

where $P_0(kT_s)$ and $P_0((k-1)T_s)$ are the position commands at the kth and the $(k-1)$th sampling times, respectively. That is, the position increment during every sampling time is the same. However, since no physical system can achieve the above rectangular velocity profile due to impulse acceleration, the acceleration interval to increase velocity from the rest to a specified value and the deceleration interval to decrease velocity from the specified value to the rest are needed.

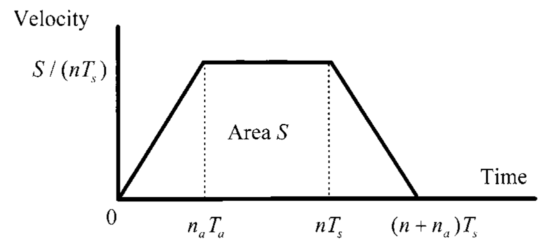

Fig. 2. A trapezoidal velocity profile for moving a distance S.

A. Selection of Polynomial Functions

Given an acceleration interval $T_a = n_a T_s$, a deceleration interval $T_d = n_d T_s$ where $n_a = n_d$, and a distance S, a trapezoidal velocity profile which has the linear acceleration and deceleration characteristics can be constructed as in Fig. 2. If this profile has the constant velocity interval, then $n = \lceil S/(V_{\max}T_s)\rceil$ is larger than n_a. The velocity and position equations for this profile are

$$V_1(t) = \begin{cases} \left(\frac{S}{nT_s}\right)\frac{t}{T_a}, & 0 \le t \le T_a \\ \frac{S}{nT_s}, & T_a \le t \le nT_s \\ -\left(\frac{S}{nT_s}\right)\left(\frac{t}{T_a} - \frac{n}{n_a}\right) + \frac{S}{nT_s}, & nT_s \le t \le nT_s + T_a \end{cases} \tag{5}$$

$$P_1(t) = \begin{cases} \left(\frac{S}{2nT_s}\right)\frac{t^2}{T_a}, & 0 \le t \le T_a \\ \left(\frac{S}{2nT_s}\right)(2t - T_a), & T_a \le t \le nT_s \\ -\left(\frac{S}{2nT_s}\right)\left(\frac{t^2}{T_a} - \left(\frac{2n}{n_a} + 2\right)t + \frac{n}{n_a}T_s + T_a\right), & nT_s \le t \le nT_s + T_a. \end{cases} \tag{6}$$

In the above derivation, it is assumed that the acceleration interval is the same as the deceleration interval and the constant velocity interval is present. In the case that the acceleration interval is different from the deceleration interval or the constant velocity interval is not present, similar equations are possible to derive. This trapezoidal velocity profile which has the linear acceleration and deceleration characteristics is particularly effective for controlling a machine tool having a high degree of rigidity. However, large jerk quantity at the stop point in the trapezoidal velocity profile may wear out a leadscrew assembly very quickly and be likely to make machine tools having little rigidity vibrate. Since smooth velocity profiles that have parabolic (second order) or higher order acceleration and deceleration characteristics do not have large jerk quantity at the stop point, these velocity profiles are very often used for industrial robots and CNC machine tools [2]–[7]. Several techniques that select a polynomial function for generating position trajectories with small jerk quantity have been proposed [1]. Given position, velocity, and acceleration at the initial and final positions, a class of polynomial functions for satisfying these conditions is selected. One approach is to split the entire trajectory into several trajectory segments so that different polynomials of a lower de-

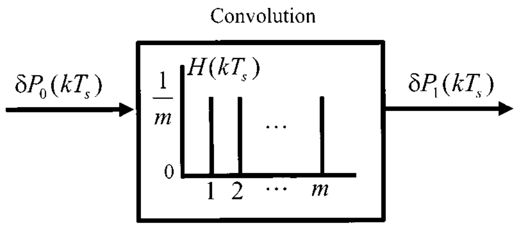

Fig. 3. A digital convolution for a trapezoidal velocity profile.

gree can be used in each trajectory segment. The most common methods are 4-3-4 trajectory, 3-5-3 trajectory, and 5-cubic trajectory. But these 4-3-4, 3-5-3, and 5-cubic trajectories can not be matched to the system limits such as the maximum velocity and the maximum acceleration. In order to match to the system limits and to have small jerk quantity, other trajectories than the above 4-3-4, 3-5-3, and 5-cubic trajectories must be generated. These position trajectories can be generated by the selection of higher order polynomial functions. The major problem in generating position trajectories by the selection of higher order polynomial functions is computational load. The required time for generating a position trajectory of which the velocity profile is smooth increases almost exponentially with added order. Therefore, it is almost impossible to use these techniques for generating a position trajectory of which the velocity profile is arbitrarily smooth because of time constraints in controlling industrial robot and CNC machine systems. Some robot/CNC companies already have used the velocity generation techniques other than the selection of polynomial functions [2]–[4], [7].

B. Digital Convolution Techniques

Given an acceleration interval $T_a = n_a T_s$ (which determines the deceleration interval $T_d = n_d T_s = n_a T_s$ because two intervals cannot be different in velocity profiles generated by digital convolution techniques) and a desired distance S, a trapezoidal velocity profile which has the linear acceleration and deceleration characteristics can be constructed by digital convolution techniques. The position increment during each sampling time in a trapezoidal velocity profile $\delta P_1(kT_s)$ is constructed by the convolution of the position increment $\delta P_0(kT_s)$ as in (4) during each sampling time in a rectangular velocity profile and the sequence $H(kT_s)$ given by

$$H(kT_s) = 1/m, \qquad 1 \le k \le m \tag{7}$$

where

$$m = T_a/T_s = n_a \tag{8}$$

as in Fig. 3. The relationship between δP_0 and δP_1 is expressed as the following recursive equation [5]:

$$\delta P_1(kT_s) = \frac{\delta P_0(kT_s) - \delta P_0((k-m)T_s)}{m} + \delta P_1((k-1)T_s). \tag{9}$$

Equation (9) provides the basic information for the trapezoidal velocity profile. Based on (9), we can design the hardware system for trapezoidal velocity profiles as in Fig. 4, where buffer registers act as delay elements [7]. A second-order velocity profile that has the parabolic acceleration and deceleration characteristics can also be constructed by digital convolution

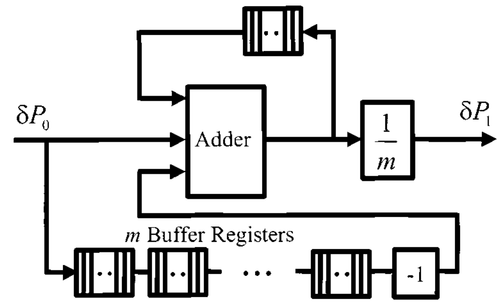

Fig. 4. A hardware structure for a trapezoidal velocity profile.

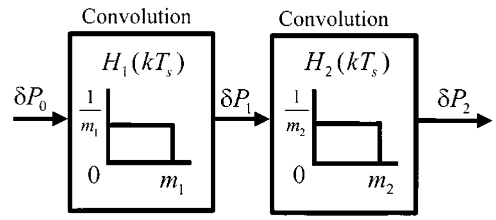

Fig. 5. Successive digital convolutions for a second-order velocity profile.

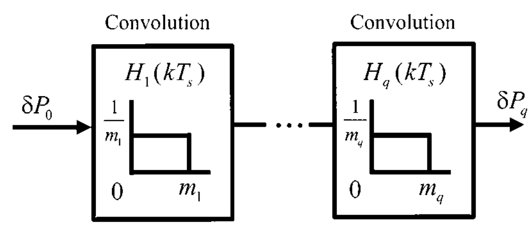

Fig. 6. Successive digital convolutions for a higher order velocity profile.

techniques. The position increment in a second-order velocity profile δP_2 can be obtained by successive convolutions as in Fig. 5 where

$$H_i(kT_s) = 1/m_i \quad \text{for } 1 \le k \le m_i. \tag{10}$$

The values of m_1 and m_2 can determine the shape of the second-order velocity profile [5], [7]. The relationship between $\delta P_1(kT_s)$ and $\delta P_2(kT_s)$ is expressed as the following recursive equations [5]:

$$\delta P_1(kT_s) = \frac{\delta P_0(kT_s) - \delta P_0((k-m_1)T_s)}{m_1} + \delta P_1((k-1)T_s) \tag{11}$$

$$\delta P_2(kT_s) = \frac{\delta P_1(kT_s) - \delta P_1((k-m_2)T_s)}{m_2} + \delta P_2((k-1)T_s). \tag{12}$$

Similarly, the position increment of a smooth velocity profile that has the high-order acceleration and deceleration characteristics can be obtained by several successive convolutions as in Fig. 6 where each sequence has the similar meaning as in (10). The shape of the smooth velocity profile can be determined by the values of $m_1, m_2, \cdots, m_q$, and q [5], [7]. The relationships between $\delta P_0, \delta P_1, \cdots$, and δP_q are expressed as the following recursive equations [5]:

$$\delta P_i(kT_s) = \frac{\delta P_{(i-1)}(kT_s) - \delta P_{(i-1)}((k-m_i)T_s)}{m_i} + \delta P_i((k-1)T_s), \qquad \text{for } 1 \le i \le q. \tag{13}$$

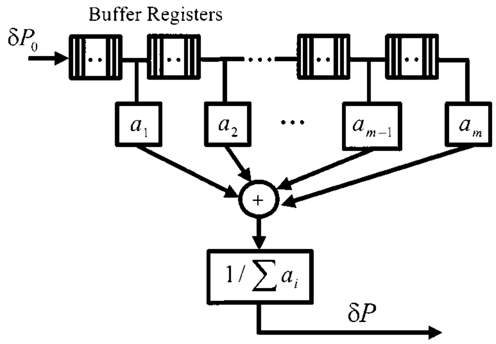

Fig. 7. The hardware structure of arbitrary acceleration and deceleration.

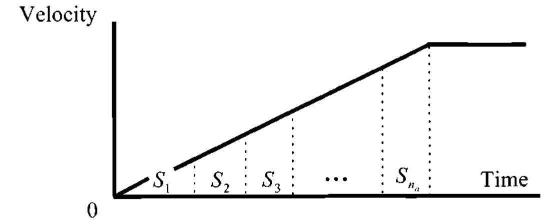

Fig. 8. The position increment during each sampling time in the acceleration interval of a trapezoidal velocity.

The velocity profiles generated by (13) have the property that the moving distances during the acceleration interval and during the deceleration interval are the same. These distances are also the same as the moving distance of trapezoidal velocity profile during the acceleration interval under the same condition.

In order to generate an arbitrary shape velocity profile, the position increment $\delta P(kT_s)$ during each sampling time in the desired velocity profile can be generated from the following convolution [5], [7]:

$$\delta P(kT_s) = \left(\sum_{i=1}^{k} \delta P_0((k-i)T_s)a_i\right) \Bigg/ \left(\sum_{i=1}^{m} a_i\right). \quad (14)$$

From the appropriate of choice a_i, for $i = 1, 2, \cdots, m$, the desired velocity profile can be obtained. The hardware system based on (14) for generating an arbitrary velocity profile can be designed as in Fig. 7. The acceleration and deceleration interval T_a is given by $T_a = mT_s$ [5], [7].

In [6], a similar convolution technique that can generate velocity profiles having several acceleration and deceleration characteristics is derived. While the computational load for generating a smooth velocity profile by these digital convolution techniques is much less than that by the selection of polynomial techniques, the deceleration characteristic generated by these techniques is determined from the acceleration characteristics. That is, the deceleration characteristic cannot be made to be independent from the acceleration characteristic by using digital convolution techniques. Therefore, some useful velocity profiles for industrial robots and CNC machine tools cannot be generated by these convolution techniques.

III. Proposed Trajectory Generation Technique

As in Section II, let us consider one single-axis control system of which the maximum velocity, the maximum acceleration, and the sampling time are $V_{\max}, A_{\max}$, and T_s, respectively. Given an acceleration interval $T_a = n_aT_s$, a deceleration interval $T_d = n_dT_s$, and a distance S, a velocity profile that has the desired characteristics of acceleration and deceleration can be constructed by the proposed technique.

A. Linear Acceleration and Deceleration

In the technique selecting polynomial functions, the velocity equation that has the linear acceleration and deceleration characteristics is calculated by (5). The coefficients of (5) may vary according to given conditions. But the ratio between the position increment during each sampling time in the acceleration interval is fixed. That is, $S_1 : S_2 : \cdots : S_{n_a} = 1 : 3 : \cdots : (2n_a - 1)$ in Fig. 8. Similarly, the ratio between each position increment in the deceleration interval is fixed. Therefore, the velocity profile that has the linear acceleration and deceleration characteristics can be calculated as follows.

1) The velocity after the acceleration V_m is determined as

$$V_mT_s = \begin{cases} \dfrac{S}{\frac{1}{2}n_a + \frac{1}{2}n_d} & \text{if } N \le 0 \\ \dfrac{S}{N + \frac{1}{2}n_a + \frac{1}{2}n_d} & \text{if } N > 0 \end{cases} \quad (15)$$

where

$$N = \left\lceil \frac{S}{V_{\max}T_s} - \frac{1}{2}n_a - \frac{1}{2}n_d \right\rceil. \quad (16)$$

N represents the interval that the velocity is constant near the maximum velocity $V_{\max}$. That is, the velocity near the maximum velocity $V_{\max}$ maintains during NT_s for positive N. If N is negative, there is no interval that the velocity is constant near the maximum velocity.

2) Then, the position increment during each sampling time is calculated as

if $N \le 0$

$$\delta P_1(kT_s) = \begin{cases} \left(\dfrac{2k-1}{2n_a}\right) V_mT_s, & 1 \le k \le n_a \\ \left(\dfrac{2(n_a + n_d - k) + 1}{2n_d}\right) V_mT_s, & n_a + 1 \le k \le n_a + n_d \end{cases} \quad (17)$$

if $N > 0$

$$\delta P_1(kT_s) = \begin{cases} \left(\dfrac{2k-1}{2n_a}\right) V_mT_s, & 1 \le k \le n_a \\ V_mT_s, & n_a + 1 \le k \le n_a + N \\ \left(\dfrac{2(n_a + N + n_d - k) + 1}{2n_d}\right) V_mT_s, & n_a + N + 1 \le k \le n_a + N + n_d. \end{cases} \quad (18)$$

According to the acceleration and deceleration interval, the coefficients $(2k-1)/(2n_a)$ and $(2(n_a + N + n_d - k) + 1)/(2n_a)$ in (17) and (18) can be calculated and be stored in advance. Therefore, a velocity profile which has the linear acceleration and deceleration characteristics can be efficiently calculated for a given distance.

B. Arbitrary Acceleration and Deceleration

Let us consider a velocity profile $V(t)$ that has the acceleration characteristic and the deceleration characteristic represented by $f_a(u)$ and $f_d(u)$, respectively

$$V(t)=\begin{cases} f_a(t/T_a), & \text{for } 0\le t\le T_a \\ V_m, & \text{for } T_a\le t\le T_c \\ f_d((t-T_c)/T_d), & \text{for } T_c\le t\le T_c+T_d \end{cases} \quad (19)$$

where both of $f_a(u)$ and $f_d(u)$ are differential on $0<u<1$ and are continuous on $0\le u\le 1$, and T_c is the time to start deceleration and V_m is the velocity after acceleration. Then, the position increment during each sampling time in the acceleration interval can be represented as

$$\delta P(kT_s)=\int_{(k-1)T_s}^{kT_s} f_a(t/T_a)\,dt = T_a\int_{(k-1)/n_a}^{k/n_a} f_a(u)\,du, \quad \text{for } 1\le k\le n_a \quad (20)$$

and it can be written as

$$\delta P(kT_s)=T_a\int_{(k-1)/n_a}^{k/n_a} f_a(u)\,du = {}^a\gamma_k V_m T_s \quad (21)$$

where ${}^a\gamma_k$ is the coefficients which can be calculated from $f_a(u)$ and $n_a=T_a/T_s$ and be stored. Similarly, the position increments during each sampling time in the deceleration interval can be represented as

$$\begin{aligned}\delta P(kT_s)&=\int_{T_c+(k-1)T_s}^{T_c+kT_s} f_d((t-T_c)/T_d)\,dt\\ &=T_d\int_{(k-1)/n_d}^{k/n_d} f_d(u)\,du\\ &={}^d\gamma_k V_m T_s\end{aligned} \quad (22)$$

where ${}^d\gamma_k$ is the coefficients which can be calculated from $f_d(u)$ and $n_d=T_d/T_s$ and be stored. T_a and T_d are not so long compared with T_s as to require much memory in most industrial robots and CNC machine tools. The area S_a under f_a during the acceleration interval and the area S_d under f_d during the deceleration interval can be represented as

$$\begin{aligned}S_a&=\int_0^{T_a} f_a(t/T_a)\,dt\\ &=T_a\int_0^1 f_a(u)\,du\\ &=\alpha_a V_m T_a\\ &=\alpha_a n_a V_m T_s\end{aligned} \quad (23)$$

$$\begin{aligned}S_d&=\int_{T_c}^{T_c+T_d} f_d((t-T_c)/T_d)\,dt\\ &=T_d\int_0^1 f_d(u)\,du\\ &=\alpha_d V_m T_d\\ &=\alpha_d n_d V_m T_s\end{aligned} \quad (24)$$

where α_a can be calculated from $f_a(u)$ and n_a and be stored. Similarly, α_d can be calculated from $f_d(u)$ and n_d and be stored. In the technique selecting polynomial functions, the velocity equation that has the acceleration and deceleration characteristics represented by $f_a(u)$ and $f_d(u)$, respectively, is calculated by appropriate polynomial functions. The coefficients of the polynomial function may vary according to given conditions.

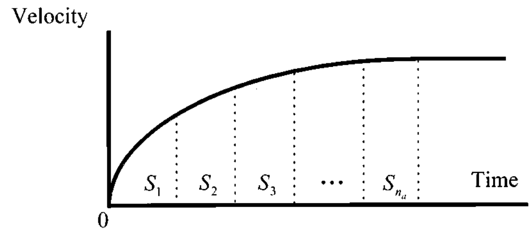

Fig. 9. The position increment during each sampling time in the acceleration interval of a velocity profile.

TABLE I
EXPERIMENTAL SETUP

Items	Specification
Sampling time	8 msec
Motor resolution	4096 pulse/rev
Maximum velocity $V_{\max}$	3000 rpm
Acceleration and Deceleration intervals	400 msec
Load	iron disk radius: 40mm thickness: 8mm

But the ratio between the position increment during each sampling time in the acceleration interval is fixed for a given acceleration interval $T_a=n_aT_s$. That is, $S_1:S_2:\cdots:S_{n_a}={}^a\gamma_1:{}^a\gamma_2:\cdots:{}^a\gamma_{n_a}$ as in Fig. 9. Similarly, the ratio between each position increment in the deceleration interval is fixed for a given deceleration interval $T_d=n_dT_s$. Therefore, a velocity profile that has the acceleration characteristic $f_a(u)$ and the deceleration characteristic $f_d(u)$ as in (19) can be generated as follows.

1) For an acceleration interval $T_a=n_aT_s$ and a deceleration interval $T_d=n_dT_s$, the corresponding coefficients ${}^a\gamma_1,{}^a\gamma_2,\cdots,{}^a\gamma_{n_a}$ and ${}^d\gamma_1,{}^d\gamma_2,\cdots,{}^d\gamma_{n_d}$ are retrieved.
2) For moving a distance S, the velocity after the acceleration V_m is determined as

$$V_mT_s=\begin{cases}\dfrac{S}{\alpha_a n_a+\alpha_d n_d}, & \text{if } N\le 0\\ \dfrac{S}{N+\alpha_a n_a+\alpha_d n_d}, & \text{if } N>0\end{cases} \quad (25)$$

where

$$N=\left\lceil \frac{S}{V_{\max}T_s}-\alpha_a n_a-\alpha_d n_d\right\rceil. \quad (26)$$

3) Then, the position increment during each sampling time is calculated as

$$\text{if } N\le 0 \quad \delta P(kT_s)=\begin{cases}{}^a\gamma_k V_m T_s, & 1\le k\le n_a\\ {}^d\gamma_{(k-n_a)}V_mT_s, & n_a+1\le k\le n_a+n_d\end{cases} \quad (27)$$

$$\text{if } N>0 \quad \delta P(kT_s)=\begin{cases}{}^a\gamma_k V_m T_s, & 1\le k\le n_a\\ V_mT_s, & n_a+1\le k\le n_a+N\\ {}^d\gamma_{(k-n_a-N)}V_mT_s, & n_a+N+1\le k\le n_a+N+n_d.\end{cases} \quad (28)$$

In order to avoid the accumulation of the computational errors in (27) and (28), the "actual" calculation of each position increment is performed as follows.

TABLE II
CHARACTERISTICS OF ACCELERATION AND DECELERATION

Characteristics	Acceleration $(0 \le u \le 1)$	Deceleration $(0 \le u \le 1)$
Linear	$f_a(u) = V_m u$	$f_d(u) = V_m(1-u)$
Symmetric smooth	$f_a(u) = \frac{V_m}{2}\left(\sin(\pi(u-\frac{1}{2}))+1\right)$	$f_d(u) = \frac{V_m}{2}\left(\sin(\pi(u-\frac{3}{2}))+1\right)$
Unsymmetrical smooth	$f_a(u) = V_m \sin(\frac{\pi}{2}u)$	$f_d(u) = \frac{V_m}{2}\left(\sin(\pi(u-\frac{3}{2}))+1\right)$

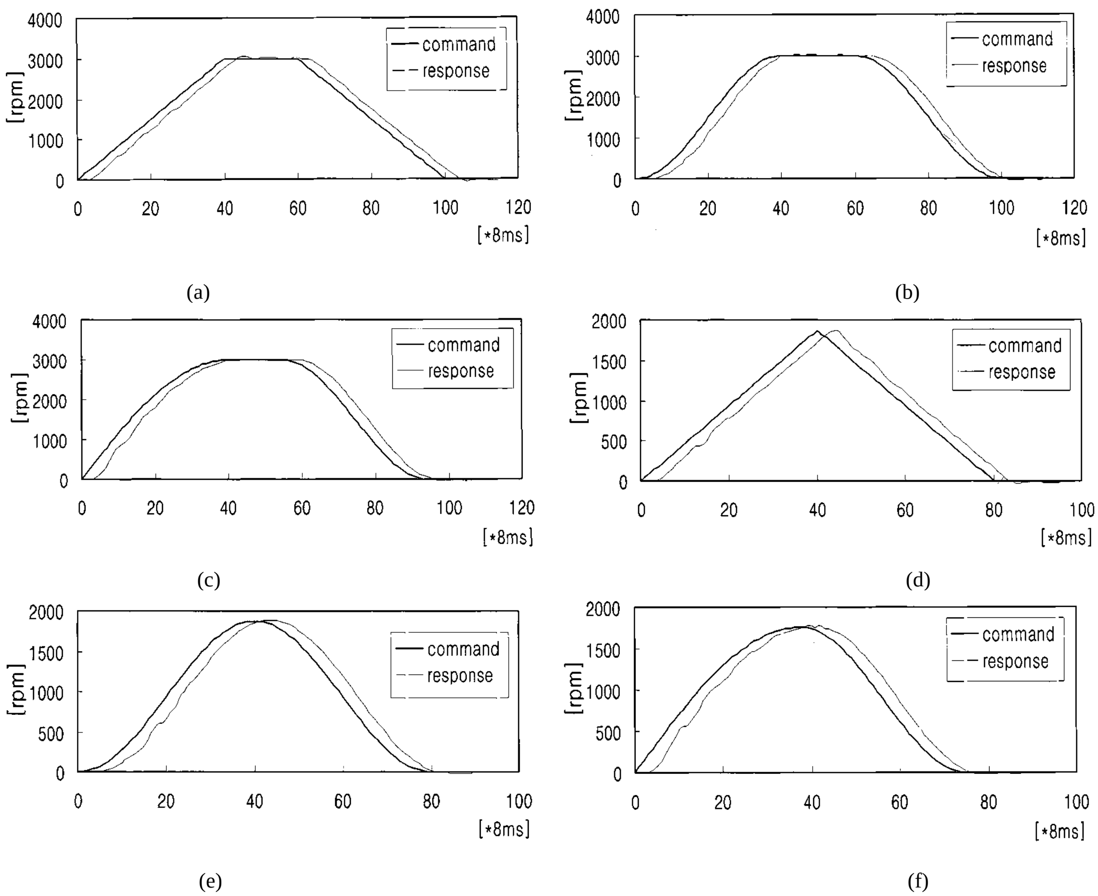

Fig. 10. Velocity commands and responses. (a) Linear acceleration and deceleration for a long distance. (b) Symmetric smooth acceleration and deceleration for a long distance. (c) Unsymmetrical smooth acceleration and deceleration for a long distance. (d) Linear acceleration and deceleration for a short distance. (e) Symmetric smooth acceleration and deceleration for a short distance. (f) Unsymmetrical smooth acceleration and deceleration for a short distance.

1) The first position increment $[\delta P(T_s)]_q$ is calculated as the roundoff value of the "true" value $\delta P(T_s)$ due to a finite accuracy computation.
2) The kth position increment $[\delta P(kT_s)]_q$ for $k \ge 2$ except the last position increment is calculated as the roundoff value of $\sum_{l=1}^{k} \delta P(lT_s) - \sum_{l=1}^{k-1}[\delta P(lT_s)]_q$.
3) The last position increment is calculated as $S - \sum(all\ previous[\delta P(kT_s)]_q$.

Since the coefficients ${}^a\gamma_1, {}^a\gamma_2, \cdots, {}^a\gamma_{n_a}$ and ${}^d\gamma_1, {}^d\gamma_2, \cdots, {}^d\gamma_{n_d}$ can be calculated and be stored in advance according to the acceleration and deceleration intervals, the velocity profile which has arbitrary acceleration and deceleration characteristics can be efficiently calculated for a given distance.

In digital convolution techniques as in (14) and Fig. 7, the shape of a velocity profile is determined by the values of a_i for $i = 1, 2, \cdots, m$ where m determines the acceleration and deceleration intervals which are same $n_a = n_d = m$. This means that the values of a_i determine the above coefficients ${}^a\gamma_1, {}^a\gamma_2, \cdots, {}^a\gamma_{n_a}$ and ${}^d\gamma_1, {}^d\gamma_2, \cdots, {}^d\gamma_{n_d}$ where $n_a = n_d$. Therefore, the coefficients ${}^d\gamma_1, {}^d\gamma_2, \cdots, {}^d\gamma_{n_d}$ in velocity profiles generated by digital convolution techniques cannot be made to be independent from the coefficients ${}^a\gamma_1, {}^a\gamma_2, \cdots, {}^a\gamma_{n_a}$. The deceleration characteristics are determined from the acceleration characteristics in velocity profiles by digital convolution techniques and some useful velocity profiles for industrial robots and CNC machine tools cannot be

TABLE III
MOVEMENT TIME FOR EACH VELOCITY PROFILE AND DISTANCE

Acceleration and deceleration of velocity profile	Long distance (24 revolutions)	Short distance (10 revolutions)
Linear	142 * 8 msec	119 * 8 msec
Symmetric smooth	135 * 8 msec	109 * 8 msec
Unsymmetrical smooth	127 * 8 msec	102 * 8 msec

generated by digital convolution techniques. In the proposed techniques, since the coefficients ${}^{d}\gamma_1, {}^{d}\gamma_2, \cdots, {}^{d}\gamma_{n_d}$ can be selected independently from the coefficients ${}^{a}\gamma_1, {}^{a}\gamma_2, \cdots, {}^{a}\gamma_{n_a}$, the deceleration characteristics can be made to be independent from the acceleration characteristics.

IV. EXPERIMENTS

As in Table I, the experimental setup consists of one brushless dc servo motor system with the load of an iron disk. Several velocity profiles as in Table II have been generated by the proposed technique and these velocity profiles have been applied to the brushless dc servo motor system. The symmetric smooth and unsymmetrical velocity profiles can be generated by the selection of polynomial functions, but with a large computational load. The unsymmetrical velocity profiles cannot be generated by the digital convolution techniques. Fig. 10 and Table III show the results of the position control under a proportional control with gain 0.45. The command velocity profiles generated by the proposed technique and its corresponding response velocity profile are represented as solid lines and dotted lines, respectively, in Fig. 10. For both movement of long and short distances, the same acceleration and deceleration intervals are selected. Thus, the velocity after the acceleration, V_m, in the long distance movement is near 3000 r/min, while that in the short distance movement is less than 3000 r/min. Under the same conditions as in Table I and the same maximum torque, velocity profiles which have the unsymmetrical smooth acceleration and deceleration characteristics have shorter movement time than those which have the linear or the symmetric smooth acceleration and deceleration characteristics. Movement time has been measured by the settling time that the position of an iron disk remains in the desired position within two pulse errors. Applying velocity profiles of the unsymmetrical smooth acceleration and deceleration characteristics to an industrial assembly robot system, the time to perform a given task can be shown to be reduced [4]. Several features of velocity profiles of various acceleration and deceleration characteristics were discussed in [5]. Depending on a given position control system, velocity profiles of an appropriate acceleration and deceleration characteristics should be selected. Other useful velocity profiles including those of which the acceleration interval is different from the deceleration interval can be generated by the proposed technique and be applied to industrial robots and CNC machine tools.

V. CONCLUSION

A generalized approach for the acceleration and deceleration of industrial robots and CNC machine tools has been proposed. Given a task in an industrial robot and a CNC system, the appropriate acceleration characteristic, the deceleration characteristic, the acceleration interval, and the deceleration interval can be determined. According to the acceleration characteristic, the deceleration characteristic, the acceleration interval, and the deceleration interval, some coefficients are calculated and are stored. By using these coefficients, a desired velocity profile for moving a given distance is calculated. Thus, an arbitrary velocity profile that cannot be generated by digital convolution techniques can be generated efficiently by the proposed technique.

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